

LABORATORY PROCEDURES IN THE STUDY OF ALGEBRA

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Degree of Master of Arts

by

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CHAPTER I

THE PROBLEM

Perhaps the most significant change in the teaching of elementary algebra within the last half century has been the redefinition of its purpose. Formerly taught to students as if they were all to become pure mathematicians, algebra is now considered in terms of its particular worth to the development of well-informed and responsible members of society. General acceptance of the principle that "reflection is the method of intelligent learning"¹ has led to an increased consideration of the means to effective instruction in all areas of learning. In algebra this has meant that an adequate instructional program must be designed not only to develop understanding and mastery of the concepts and relationships but also to make the most significant contribution possible to students' abilities to do careful and accurate thinking.

Algebra is important in the education of the individual as it becomes for him a means for interpreting and understanding change in the world about him. In the words of John Everett, "Algebra is the science which typifies and parallels the relationships of changing values

¹John Dewey, Democracy and Education, p. 180.

which every person must recognize and deal with every day if he is to be a contributing . . . member of society."²

The effort to evolve a functional program well adapted to the education of the masses has brought about significant changes in the content and instructional techniques of algebra. Frequent criticisms have insisted that algebra has retained much of the formalism of content and organization that characterized its introduction into the American school system in the early part of the 18th century; and further, that algebra is offered to high school students without provision for the experiences necessary to develop real understanding of the concepts of algebra.

In an address in 1901 before the British Mathematical Association, John Perry heralded the importance of teaching students through their own experiments. He defended before the Association his belief that the teaching of mathematics must be reorganized to include laboratory techniques of investigation and inquiry before mathematics would function effectively in students' thinking. In his words: "what he (the student) discovers for himself, that is of real value to him, that becomes permanently part of his mental

²John P. Everett, "Algebra and Mental Perspective," Seventh Yearbook of the National Council of Teachers of Mathematics, p. 26.

machinery . . . I feel that throughout one's whole mathematical course it is important to teach a student through his own experiments, through concrete examples worked out by him."³

Professor E. H. Moore in the following year delivered before the American Mathematical Society an address in which he, in agreement with Perry, advocated "a thoroughgoing laboratory system of instruction . . . a principal purpose being as far as possible to develop on the part of every student the true spirit of research, and an appreciation, practical as well as theoretic, of the fundamental methods of science."⁴

Concurring with Perry and Moore, later studies in mathematics education have indicated the advisability of introducing concepts through students' experiences with concrete materials.

The Committee on Mathematical Requirements, in 1923, indicated that increased emphasis should be placed on the development of an ability to grasp and utilize processes and principles in the solution of concrete problems, rather than on manipulative skill.⁵ The 1940 Report of the Joint Commission of the American Mathematical Association

³John Perry, Discussion on the Teaching of Mathematics, p. 8.

⁴E. H. Moore, "On the Foundations of Mathematics," First Yearbook of the National Council of Teachers of Mathematics, pp. 49ff.

⁵National Committee on Mathematical Requirements, The Reorganization of Mathematics in Secondary Education, p. 11.

and the National Council of Teachers of Mathematics observed that there is now a definite trend toward leading pupils into new topics through their own experiences.⁶ The same year the Committee on the Function of Mathematics in General Education accepted the position that "the mathematics curriculum may be built by studying concrete problem situations which arise in connection with meeting needs in the basic aspects of living."⁷ Similarly, The Commission on Post-War Plans issued in 1945 their report which advocated that meanings should grow out of experience, "as that experience is analyzed and progressively reorganized in the thinking of the learner."⁸

Reorganization in the content of algebra has paralleled recommendations for teaching techniques. The several reports in mathematics have suggested that the worthwhile material of the introductory course in algebra can be organized around a few major central objectives: (1) the language and ideas of algebra; (2) the fundamental skills and techniques; (3) the formula; (4) the graph;

⁶Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, The Place of Mathematics in Secondary Education, p. 40.

⁷Progressive Education Association, Mathematics in General Education, pp. 72f.

⁸National Council of Teachers of Mathematics, "The Second Report of The Commission on Post-War Plans," Mathematics Teacher, XXXVIII (May, 1945), p. 201.

(5) the equation; and (6) their applications to problem-solving. Mr. Betz, specialist in mathematics, in recommending these same objectives, wrote:

If the entire course is based on such a small body of objectives, we shall not only achieve a far greater economy or concentration, but shall also remove the prevailing impression that the customary course is an aimless array of isolated and irrelevant details.⁹

Where formerly mathematics instruction dealt largely with abstractions, the emphasis now is upon developing a system of thought which will help the student to reason carefully and accurately in the problem-solving process.

Raleigh Schorling in his discussion of trends in mathematics during the last quarter century observes, "There is specific emphasis to provide a greater amount of realism. In the modern class we try to keep meanings ahead of symbolism. The definitional approach . . . of an earlier day has been abandoned."¹⁰

Statement of Problem. In the light of the recommendations for leading pupils into the topics of algebra through their own experiences, the problem of this thesis is to describe the laboratory method of teaching elementary algebra, and to indicate procedures and materials which may be used to enrich the teaching of the basic concepts.

⁹William Betz, "Whither Algebra?" Mathematics Teacher, XXIII (February, 1930), p. 119.

¹⁰Raleigh Schorling, "Trends in Junior High School Mathematics," Mathematics Teacher, XXV (August, 1942), p. 1.

Related Studies. In addition to the recommendations of the aforementioned studies undertaken by educational groups, namely, the Report of the National Committee on Mathematical Requirements, the Joint Commission Report, the Progressive Education Report, and the Report of The Commission on Post-War Plans, there have been several individual studies which point to the need for a reorganization of the teaching procedures of mathematics, and in particular algebra, to include greater attention to a laboratory method of teaching.

The results of an experiment by John Ramseyer, "A Study of the Use of Laboratory Activities in High School Mathematics," suggest that the use of the laboratory has been effective in learning mathematical principles and conclusions, and in his words: "Since all of the subject matter from which these conclusions might be reached was furnished by the laboratory it is reasonable to conclude that the laboratory has been influential in this regard."¹¹

F. H. Gorman, interested in the laboratory approach to teaching, sent out questionnaires to forty-six "recognized authorities on the teaching of elementary and secondary

¹¹John A. Ramseyer, "A Study of the Use of Laboratory Activities in High School Mathematics," pp. 83f.

mathematics"¹² who were requested to indicate their opinions of the relative importance of seventy-five items of classroom equipment for mathematical use.

Of significance is his analysis of his results:

"This survey of authoritative opinion regarding items of equipment considered useful in teaching elementary and secondary school mathematics clearly indicates that the authorities in the teaching of mathematics highly recommend the use of a large number."¹³

Verna Newman in her study, "The Teaching of Inductive Reasoning Through the Channel of Algebra," emphasized the development of the concepts and principles of directed numbers, linear equations, exponents, and relationships through the use of instructional guide sheets intended to provide for transfer of reasoning to an analysis of life situations.¹⁴

Gerald Kackly made a study of "Thinking in Ninth-Grade Mathematics," in which he sought to develop habits of reflective thinking through the use of a series of written exercises. Of interest is his finding: "Nature of proof ideas cannot be used in exercises to teach pupils to think."¹⁵

¹²F. H. Gorman, "What Laboratory Equipment for Elementary and High School Mathematics?" p. 335.

¹³Ibid., p. 344.

¹⁴Verna Newman, "The Teaching of Inductive Reasoning Through the Channel of Algebra."

¹⁵Gerald Kackly, "Thinking in Ninth-Grade Mathematics," p. 101.

In his discussion of "The Fundamental Skills of Algebra," John P. Everett placed an emphasis on the development of the "associative skills of interpretation," rather than on computational skills.

In the high school the pupil is definitely taking on a philosophy of life which either restricts his mental vision or enables him to emerge from the naive world of sensations into a new universe of thought where laws of quantitative measurement and organization of measurable forces give to events and things meanings never revealed by their mere existence The child needs algebra for what it will enable him to do, but infinitely more does he need it for the orderly way in which, in the midst of constant change, it will enable him to think.¹⁶

Another study related to learning in algebra, "Building Algebraic Concepts Through the Process of Induction," is reported by James Despinasse in which he attempts to provide study guides, field and laboratory work, resource units, texts, and publications for use in building algebraic concepts through the inductive process.¹⁷

Mary L. Webster in 1943 completed a study in which she discussed the characteristics of symbolism and provided materials designed to make pupils conscious of the prevalence and value of symbolism in their daily lives.¹⁸

¹⁶John P. Everett, "The Fundamental Skills of Algebra,"

¹⁷James Despinasse, "Building Algebraic Concepts Through the Process of Induction."

¹⁸Mary L. Webster, "Elementary Algebra with an Emphasis on Symbolic Thinking."

Organization of Remainder of Thesis. In this introductory chapter discussion has centered upon the change in the fundamental purpose of algebra as it has been taught in the secondary school, the recommendations for laboratory procedures as a desirable method for developing reflective habits of thinking, and finally a discussion of previous studies concerned with algebra and the laboratory method.

The laboratory method is defined in Chapter Two and its purposes stated. Discussion is given the outcomes expected from the use of this method. Many leaders in education are agreed that content and method of mathematics teaching must be changed considerably from the more formalized and traditional approach. A review of the views of these educators completes the chapter.

In Chapter Three a detailed discussion of classroom materials and equipment, and suggestions for their use, is followed by a source list of materials useful in the study of algebra.

Included in Chapter Four are study guides for student activities leading to the development of meanings and understandings in the study of formulas, graphs, equations, and directed numbers. As samples of directed activities in algebra, they are suggested as possibilities of the work in the laboratory, and are designed to give pupils maximum experience in investigating and discovering the laws and principles of algebra.

Chapter Five, the final chapter, summarizes and offers a brief review of the procedures and materials presented.

CHAPTER II

THE LABORATORY METHOD

Definition. In contrast to earlier methods of merely passing on information, the laboratory method of teaching is designed to stimulate activity and discovery on the part of the learner. Designed for investigation and discovery through actual participation by the student, the laboratory method emphasizes education on an active, direct, experimental level. Its primary objective is to help pupils to develop new concepts and meanings with greater understanding and insight through experimental activities in concrete situations. It provides stimulating and worthwhile experiences to develop meanings, to clarify understanding of principles, and to increase student interest in theory and applications. The principles and processes of measuring, counting, estimating, comparing, analyzing, collecting and organizing data from concrete physical situations, when seen in relation to actual applications, become more functional and meaningful. Supplementary instruments, devices, experiments, activities, and other materials for enrichment of the meanings of mathematics introduce, build up, and clarify abstract objects; they stimulate further activity on the part of the learner.

The laboratory approach lends itself to an experimental-intuitive treatment of the introductory

topics of algebra. The mathematics classroom becomes a laboratory for research and investigation. Through their own experiments with tangible materials, students discover, observe, derive, and generalize from concrete experience abstract laws and principles. In algebra this means the provision of materials and opportunities for students to discover the mathematical laws and principles governing relationships operating in concrete instances.

The laboratory method has value in its flexibility which permits students to work experimentally as individuals or in groups. Through their own experiments students feel that they are studying the topics themselves, and not mere words written by some authority on the subject. The development of an ability to think for themselves, to understand quantitative relationships, to analyze and judge results, aids their understanding of mathematical relationships and gives them an awareness of the interrelationships of society.

Pupil activities are specifically designed to develop effective habits of thought. Mathematics is thus viewed as a method of thinking into which pupils must grow. Content and method are interrelated in such a way that pupils acquire both the knowledge and the habits which will enable them more effectively to understand and control the elements of their environment.

The application of the principles of mathematics to the types of situations that cause reflection in ordinary life gives opportunity for careful and accurate thinking.

Students' increased insight into their environment helps them realize the interaction between mathematics and the various aspects of living.

Purposes. The primary emphasis of the laboratory method of teaching is the development of meaningful generalizations by furnishing the concrete experiences necessary for the understanding of abstract meanings.

Abstractions have been defined as generalizations that grow out of concrete experiences. They eliminate the awkwardness of using concretes in our thinking. To quote Professor J. W. A. Young, "From the point of view of the laboratory method the pupil, when weighed down by the burden of many similar concrete or numerical cases, may be easily led to see that they can all be replaced by a single . . . case. He thus abstracts his own mathematics."¹

The laboratory method is itself no guarantee however, that abstraction will emerge--it merely supplies the situation by which abstraction becomes possible and meaningful. Mere concrete experience may remain isolated and unrelated. For the emergence of generalizations, the numerous instances of concrete experience must be related and this relationships must be apparent to the learner. In the words of John Dewey: "The measure of the value

¹J. W. A. Young, The Teaching of Mathematics in the Elementary and Secondary School, p. 104.

of an experience lies in the perception of relationships. It is this discernment that is the genuinely intellectual matter."²

Well-conducted laboratory work can do much to supplement and enrich mathematical study, but unless it is carefully planned, supervised, and guided toward definite ends, it may easily degenerate into more or less aimless playing. However, with the necessary planning and accumulating of material, this type of work has definite values to contribute to the study of algebra. Some of these may be listed as:

1. An enlarged understanding of the basic concepts and principles of mathematics
2. A knowledge of the methods of expressing mathematical relationships, and an ability to use these methods
3. The development of habits of clear and accurate thinking; and of an ability to conduct experiments: to collect, arrange, and identify data, and to interpret the data so organized in terms of relations that exist between them
4. An understanding and appreciation of the many applications of mathematics
5. An increased skill in using certain instruments and devices
6. An appreciation of the historical significance of mathematics

The fulfillment of these objectives becomes apparent in the development of the remainder of this thesis.

²Dewey, op. cit., p. 169.

Views of Educators. Experimentation is the keynote of laboratory procedures. "Our best examples of functional relationship and correlation come through the study of concrete examples of physical change."³

According to the guiding principles of John Dewey, the first approach to any subject, if thought is to be aroused and not words acquired, is to call to mind the sort of experiences that interest and engage activity in ordinary life; to give the pupils something to do which arouses thinking--"something, in other words, presenting what is new (and hence uncertain or problematic) and yet sufficiently connected with existing habits to call out an effective response."⁴

An ounce of experience is better than a ton of theory simply because it is only as an experience that any theory has vital and veritable significance. An experience, a very humble experience, is capable of generating and carrying any amount of theory, but a theory apart from an experience cannot be definitely grasped even as a theory.⁵

William Betz puts this another way when he writes:

All school work should be based on the pupils' personal and immediate experience. To be sure there is universal agreement today that nothing can take the place of the child's first-hand direct contact with the world of reality. All basic meanings must be built up in that way.⁶

³Herbert R. Hamley, Relational and Functional Thinking in Mathematics, p. 21.

⁴Dewey, op. cit., p. 181.

⁵Ibid., p. 169.

⁶William Betz, "The Necessary Redirection of Mathematics," Mathematics Teacher, XXXV (April, 1942), p. 150.

The Commission on Post War Plans⁷ advocates that the first encounters with meanings should ordinarily occur in concrete situations of large personal significance to the learner and, in a later section of the same report, "Teachers of algebra must utilize wherever possible laboratory or investigational techniques and seek to give the mathematical classroom the furniture, equipment, and atmosphere of a workroom."⁸

The National Committee on The Reorganization of Mathematics maintained that "the idea of relationship or dependence between variable quantities be imparted to the pupil by the examination of numerous concrete instances of such relationships" in order to insure recognition of relationships in real life.⁹

The Progressive Education Association criticizes the tendency to impose arbitrarily certain general and abstract concepts upon the student "without his having had any responsible part in the gradual process of generalization and abstraction from concrete and specific instances arising in problems real to him."¹⁰

⁷National Council of Teachers of Mathematics, "The Second Report of the Commission on Post-War Plans," op. cit., p. 201.

⁸Ibid., p. 208.

⁹The National Committee on Mathematical Requirements, op. cit., p. 64.

¹⁰Progressive Education Association, loc. cit.

In considering the laboratory method of teaching, Professor Young writes:

The laboratory method proposes that the experimental origin of mathematics be fully recognized; that the pupil be led to feel the need of the mathematical tool through some material experiment he has made or things he has done . . . Experiment, estimation, approximation, observation, induction, intuition, common sense are to have honored places in every mathematical classroom in which the laboratory method has sway.¹¹

The word "laboratory" suggests exploration and investigation--on a direct, personal basis. It is a place where students experiment to find answers to their own questions. They study with a purpose. Learning is not a memorization of what someone else has already found out, it is an active trying-out process. The implications of such experiences are revealed in the following statement by John Dewey:

An individual is not original merely when he gives to the world some discovery that has never been made before. Everytime he really makes a discovery, even if thousands of persons have made similar ones before, he is original. The value of a discovery in the mental life of an individual is the contribution it makes to a creatively active mind; it does not depend upon no one's ever having thought of the same idea before. If it is sincere and straightforward, if it is new and fresh to me or to you, it is original in quality, even if others have already made the same discovery. The point is that it be first-hand, not taken second hand from another.¹²

¹¹Young, loc. cit.

¹²John Dewey, Construction and Criticism, p. 4, cited by John L. Childs, Education and the Philosophy of Experimentalism, pp. 185-186.

CHAPTER III

INSTRUCTIONAL MATERIALS

The older courses in mathematics required little in the way of laboratory materials. Perhaps therein lay their weakness, for there was consequently little stimulus to relate the principles of mathematics to life.¹ The success of the laboratory emphasis in teaching is dependent upon the extent to which equipment is used to give concreteness and greater reality to mathematics.

Perhaps the most outstanding feature of a modern classroom for mathematics is its resemblance to a laboratory. Equipped with bulletin boards, pictures and posters, a section of graph board, instruments, models, cupboards, file cabinets, its very personality suggests activity and informality--sharp contrast to the classroom whose entire equipment consisted of chalk, erasers, yard stick, perhaps a blackboard compass, and a few dusty pictures of ancient mathematicians.

Planning the Classroom. Adapting the classroom for laboratory instruction involves planning to include the instruments and materials appropriate to the activities--as well as to insure the provision of the more general

¹The National Committee on Mathematical Requirements, op. cit., p. 277.

features attributed to a good classroom. Flexible furniture is indispensable, and flat-top tables with comfortable chairs serve very satisfactorily. The room should contain adequate blackboard space--including at least one coordinate blackboard section. There should be blackboard equipment consisting of white and colored chalk, blackboard protractors, compasses, pointers, meter stick, and yardstick. The value of a demonstration table for setting up experimental apparatus may be readily seen. Bulletin boards, bookshelves, magazine racks, chart files, storage space for instruments and equipment, files for completed assignments and evaluation materials, and display space for models and instruments are all desirable features of the mathematics laboratory.

A center of interest, as well as an essential feature of the room, is the mathematical library in which are to be found supplementary reference books in mathematics and related subjects as well as books and magazines for recreational reading, to enhance the appreciation of mathematics. Appropriate pictures relating to its history, progress, and application add to the general attractiveness of the room and provide a favorable atmosphere for the stimulation and maintenance of a high degree of student interest.

Figures 1, 2, 3, 4, and 5 illustrate these desirable characteristics and equipment in a well-planned mathematics classroom.

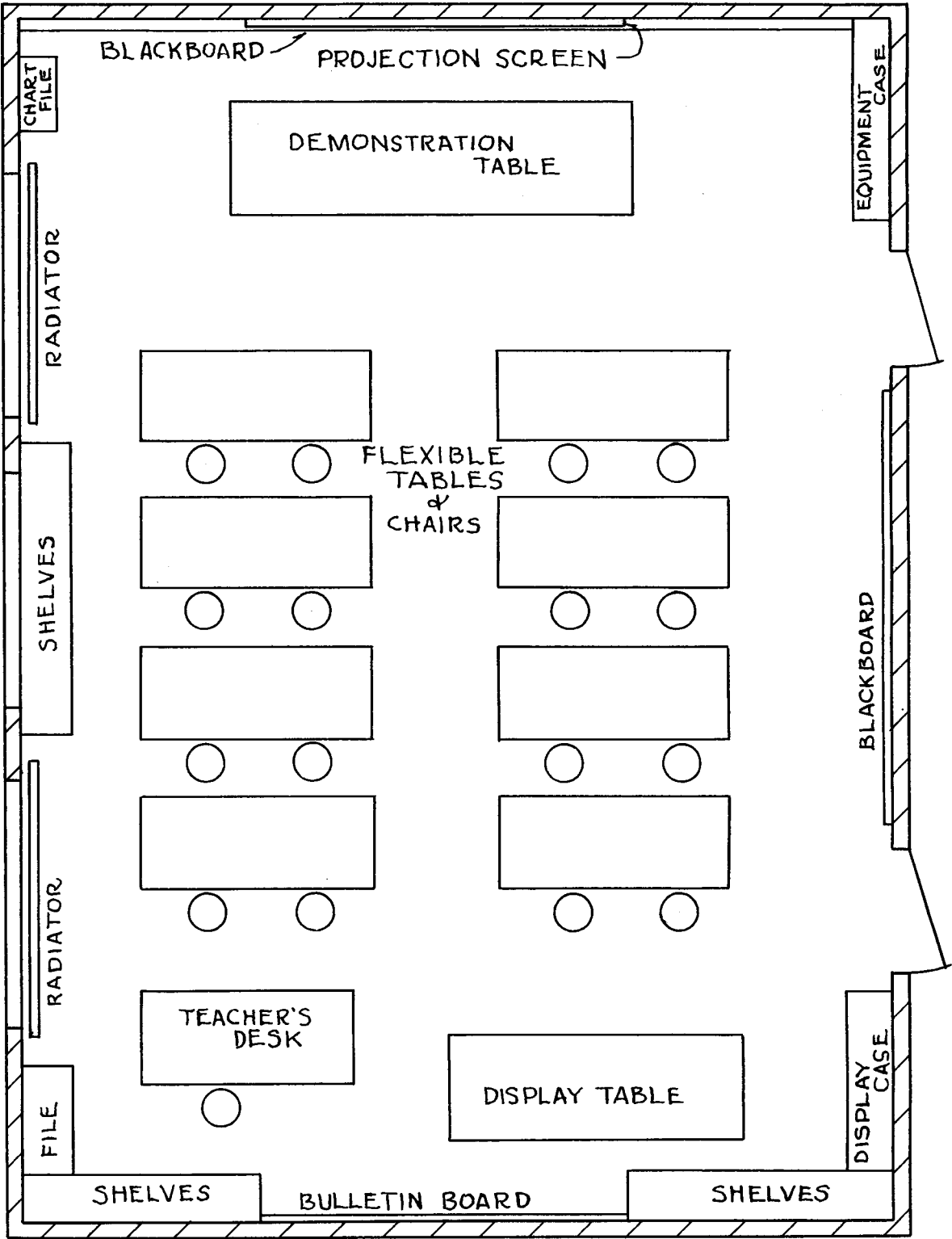


Figure 1 - Floor Plan of a Mathematics Laboratory

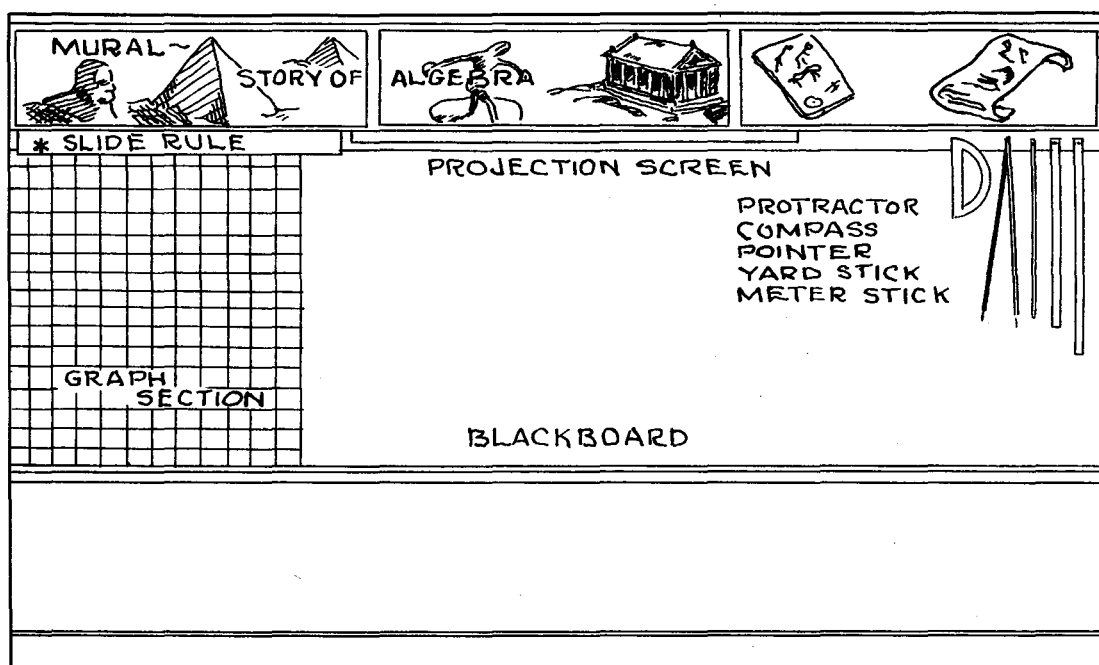


Figure 2 - Front Wall Elevation

*Rack for slide rule or number
scale whichever is currently in use

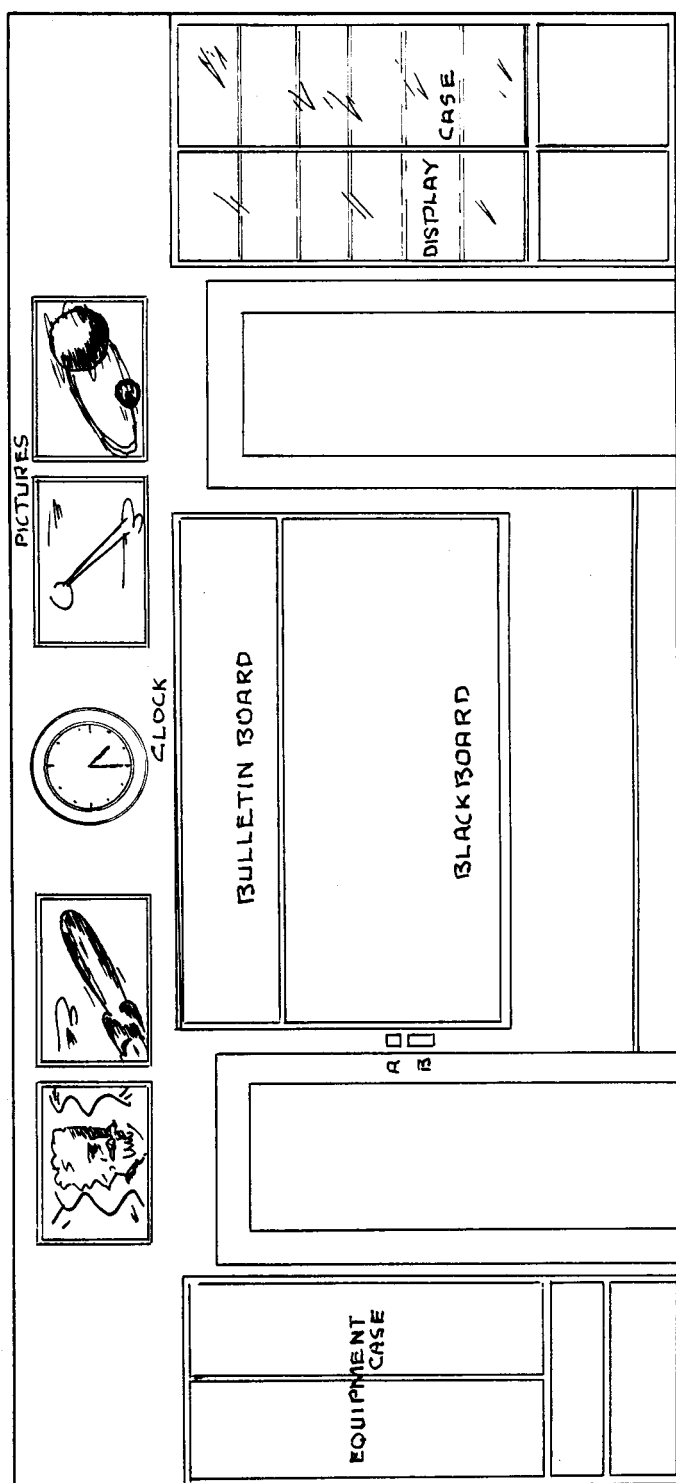


FIGURE 3- INSIDE WALL ELEVATION

- A. LIGHT SWITCH
- B. PENCIL SHARPENER

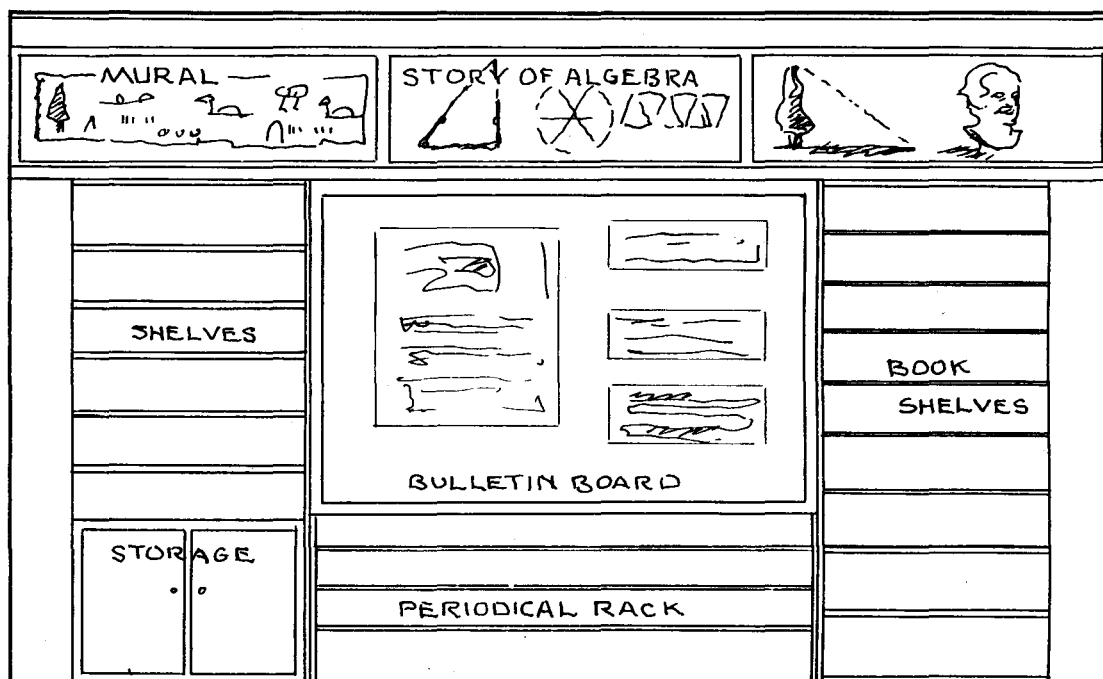


Figure 4 - Rear Wall Elevation

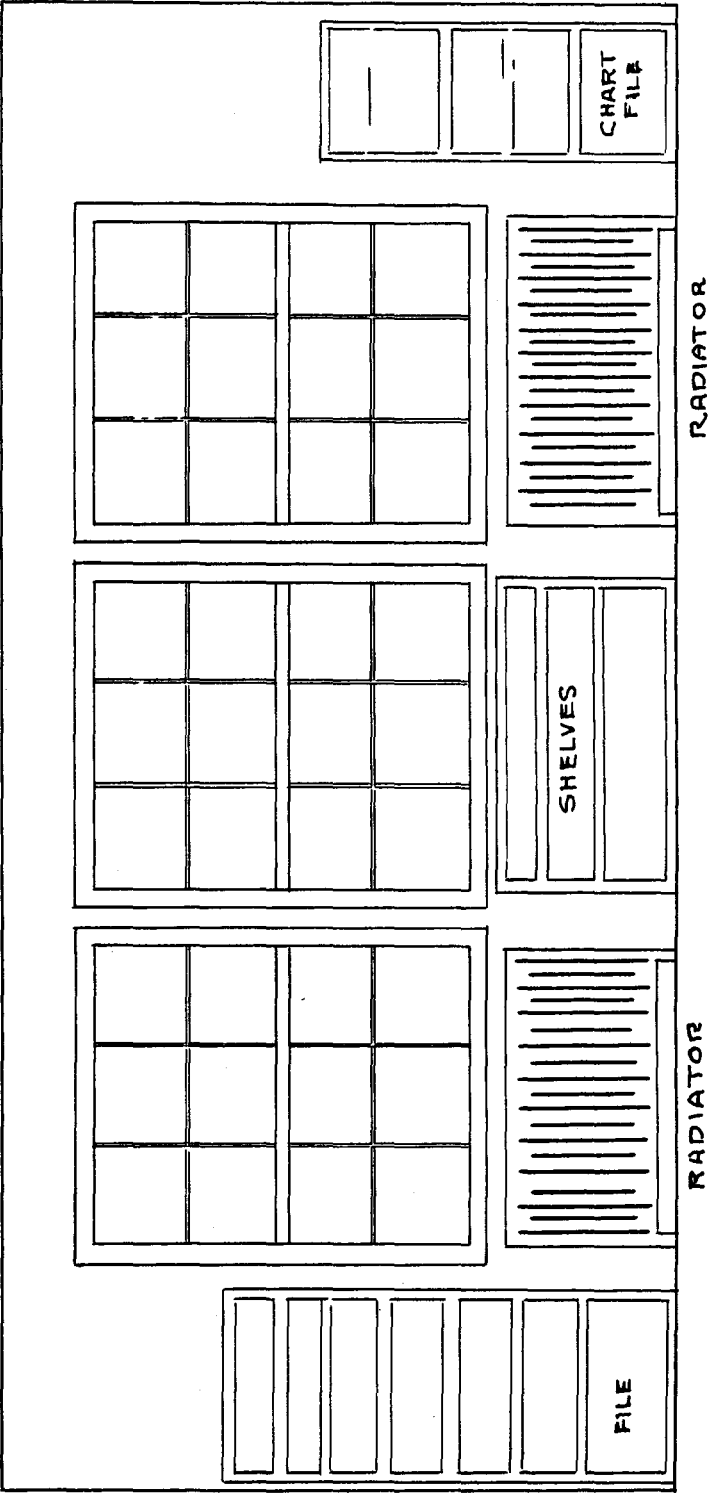


FIGURE 5- OUTSIDE WALL ELEVATION

Instruments and Devices. Instruments particularly useful for developing concepts in algebra range all the way from rather elaborate models bought from commercial manufacturing companies to simple mock-ups made by students themselves. Though the devices made by students are not as precisely constructed as commercially manufactured instruments, the results obtained are well within the scope of classroom purposes. Students derive much satisfaction in the actual construction of instruments and are more likely to understand the mathematical principles upon which these instruments are based. To illustrate, a mock-up of a directed number scale similar to the one shown in Figure 6 is invaluable for experimentation in understanding and developing the concept of directed numbers.

Simple and inexpensive to construct, a number scale may be made by securing a dime-store slide rule, sand-papering off the graduations, repainting, and numbering the scale as illustrated. The value, as mentioned above, of the student's direct participation in the construction of an instrument of this kind lies not only in its use for deriving the laws and principles of operation, but more significantly in its contribution to his greater understanding of the meaning of directed numbers.

An adjustable triangle for investigation of the general properties of triangles and for developing a

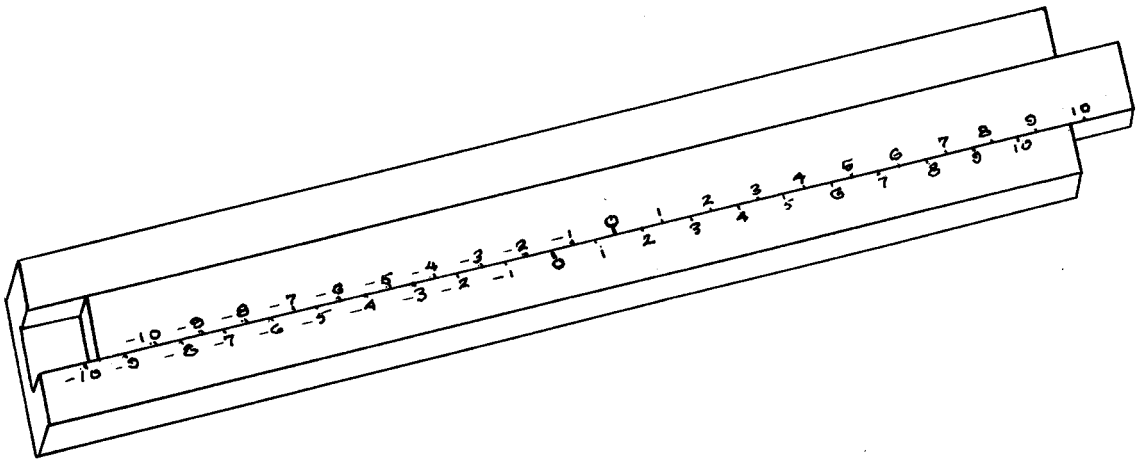


Figure 6 - Directed Number Scale

general algebraic statement for the sum of the angles of a triangle can be easily constructed from sheet metal or thin plywood. (See Figure 7.) Each of three sides of any desired length is laid out on the sheet material. The base is equipped with semicircular protractors at each end as in Figure 8. Small protractors may be cemented on to the flat base member; or the angular graduations may be marked on the member itself, using a commercial protractor as a pattern. Holes for fastening are drilled at the center point of each protractor. One of the side members is also provided with a protractor end, the opposite end being slotted for a length of several inches. The third side member is slotted for a slideable joint at A. The other end is drilled for a pivoting fastening at C. By joining the members with small screws, brads, or rivets, at points A, B, and C, a flexible triangle is provided--adjustable to virtually any angular measurements.

Many plane figures of various sizes for use in developing area and perimeter formulas, may be constructed from sheet materials such as enameled cardboard, plywood, or sheet metal. Among these devices are discs, triangles, parallelograms, rectangles, and sectors of circles. The simple construction involves little more than laying out figures of the desired dimensions and cutting them out. Of use in development of area formulas are square units of measure cut out in similar manner.

Figure 7 -
Adjustable Triangle
(Assembled)

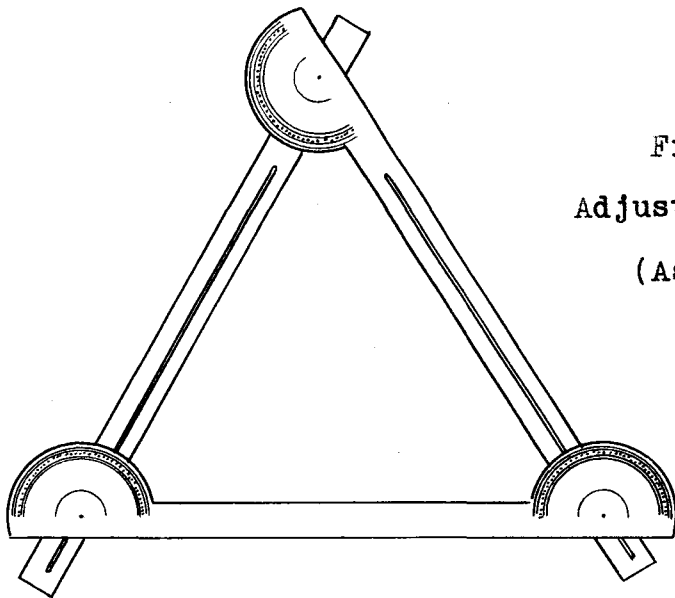
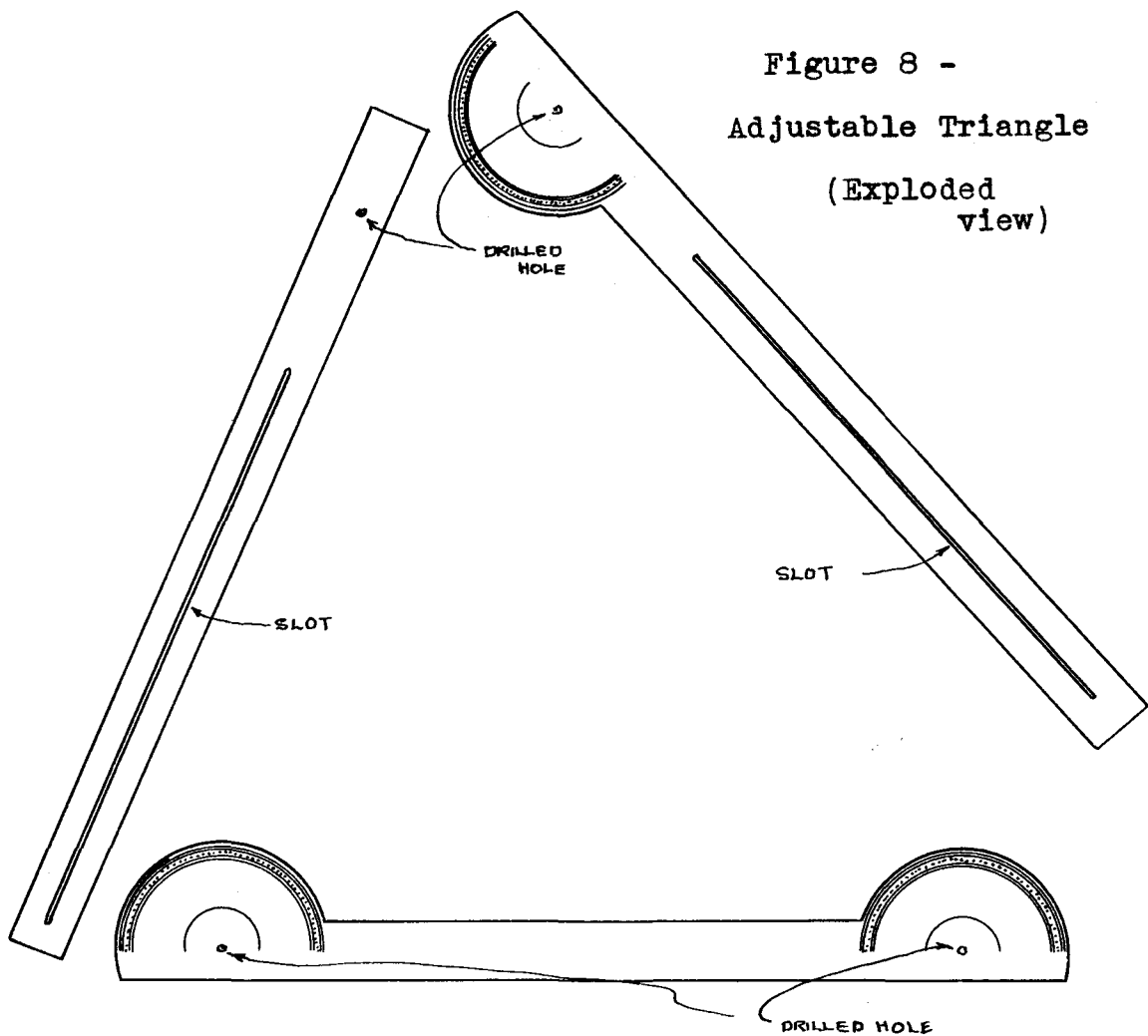


Figure 8 -
Adjustable Triangle
(Exploded view)



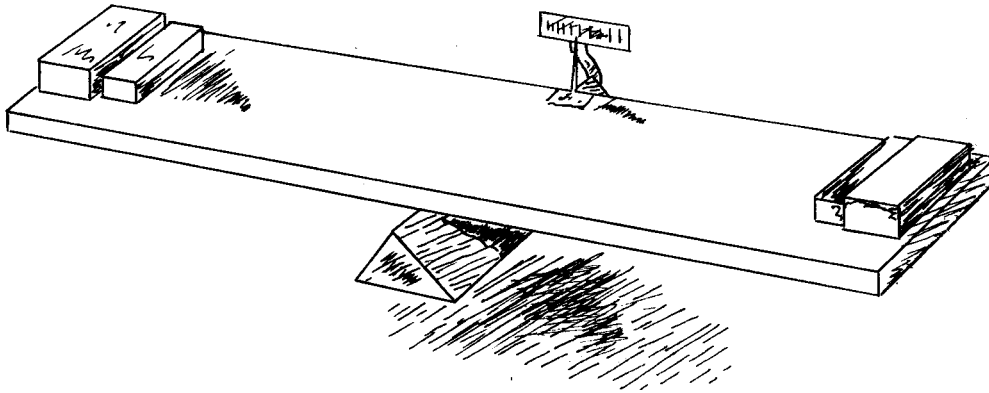


Figure 9 - A Simple Balance

A simple balance or scale is recommended for use in developing the laws that govern equations. This instrument is an essential part of mathematics laboratory equipment. Although balances are economically available commercially, an effective device for this purpose may be made by the students. A wedge-shaped wooden fulcrum provides the support for a flat wooden balance, as in Figure 9. Duplicate sets of weights may be cut from wooden blocks.

Reference Materials. Increased interest in the principles and processes of algebra stimulates students' desires to locate more information on particular topics. They may want to compare authors' viewpoints on the interpretation of meanings; or to find current applications of algebraic topics. Historical and biographical material appeals strongly to certain students. Individual research, reports to the class, or group projects to locate information, all reveal to the student how closely the development of mathematics is linked to that of civilization.

Reference materials provide students with a background of understandings. Pamphlets, texts, essays, and related popular non-fiction material provide pertinent and valuable information in the study of algebra.

Obviously, it would be impractical here to list all of the many materials that are available. There is included however a sample list of books illustrative of

the type that may prove useful in the laboratory.

History of Mathematics

Sanford, Vera. "A Short History of Mathematics."
Boston: Houghton Mifflin Company, 1930.

Smith, David Eugene. "History of Mathematics,"
(Two Vols.) New York: Ginn and Company, 1925

Supplementary Texts

Breslich, Ernst R. "Algebra An Interesting Language."
New York: Newson and Company, 1939.

Milne, William J. and Walter F. Downey. "Activities
in Algebra." New York: American Book Company, 1937.

Schorling, Raleigh and John R. Clark. "Mathematics
In Life--Formulas and Equations." Yonkers-on-
Hudson, New York: World Book Company, 1936.

Schorling, Raleigh and John R. Clark. "Mathematics
in Life--Uses of Graphs." Yonkers-on-Hudson, New
York: World Book Company, 1936.

Applications of Mathematics

Axelrod, Aaron. "Machine Shop Mathematics." New
York: McGraw-Hill Company, 1944.

Brinton, W. C. "Graphic Presentation." New York:
Brinton Associates, 1939.

Felker, C. A. "Shop Mathematics." Milwaukee,
Wisconsin: The Bruce Publishing Company, 1943.

Lasley, Sidney J. and Myrtle F. Mudd. "The New
Applied Mathematics." New York: Prentice-Hall,
Inc., 1937.

Items appearing in current magazines and newspapers offer a wide variety of materials for classroom discussion and use. For example, a striking graph clipped from a recent newspaper or magazine has far greater appeal and application than a number of graphs in a textbook. This graph is current material--tangible evidence that graphs

are in daily use to picture relationships, and that an ability to read, interpret, and even make graphs is a necessary part of the equipment of an intelligent person. Illustrations brought in by students themselves cause the formation of a growing conviction that mathematics is an important aspect of the social life of the community of which they are a part.

An abundance of information is furnished in graphs, tables, and verbal statements. Typical of the wide variety of subjects that find expression in graphical and tabular form are:

1. Temperature graphs
2. Temperature tables
3. Baseball scores
4. Popularity polls
5. Athletic team standings
6. Population trends
7. Production rates
8. Taxes
9. Stock quotations
10. Racing results
11. Vital statistics
12. Statements of assets and liabilities
13. Corporate gains and losses
14. Batting averages
15. Price variations
16. Bond sales
17. Statements of earnings
18. Treasury statements
19. Election trends
20. Exports and imports

The issues of "Building America," a periodical published by the Society for Curriculum Study, Inc., 425 West 123rd Street, New York, offer excellent graphs of current conditions. The World Almanac as well as encyclopedias contains much information which may be adapted to the local situation.

Graphic Materials. Clippings and pictures from current periodicals, charts of the history and progress of algebra, posters of algebraic uses and applications, portraits and biographical sketches of mathematicians, diagrams and cartoons related to algebra form the bulk of the materials classified as graphic. Though graphs are essentially graphic materials, because of their major importance to algebra they were considered earlier in the discussion of reference materials.

Many and varied suggestions for material of this type are listed in Woodring and Sanford's "Enriched Teaching of Mathematics in the Junior and Senior High School."² A source-book for teachers, listing free and low-cost illustrative and supplementary materials, it includes items related to formulas, directed numbers, graphs, verbal problems, progressions, logarithms, and the slide rule. Sections are also devoted to sources of historical and recreational materials, equipment, pictures, and periodicals.

Additional material that will prove helpful in motivating mathematics includes a free wall-size chart issued by Scott Foresman and Company, 120 East 23rd Street, New York, entitled "High-lighted History of Algebra."

²Woodring, Maxie, and Vera Sanford. Enriched Teaching of Mathematics in the Junior and Senior High School. New York: Bureau of Publications, Teachers College, Columbia University, 1938.

Besides showing the leading contributions of famous mathematicians, the chart pictures algebra's slow development from the simple equations of Ahmes, 3600 years ago, down to its modern form. Only key points are touched upon, but they provide background notes for an approach to such topics as the equation, graphs, positive and negative numbers, fractions, quadratics, and others.

The Educational Service Department of the Chevrolet Motors Division, General Motors Sales Corporation in Detroit, issues a chart titled: "Illustrations Depicting the Use of Mathematics in Motor Car Engineering." This chart illustrates the uses of arithmetic, algebra, trigonometry, and geometry in the manufacture of the motor car.

Scripta Mathematica, 186th Street at Amsterdam Avenue, New York, publishes two portfolios of portraits of eminent mathematicians. The portraits are each size 10" x 14" and suitable for framing. David Eugene Smith's biographical sketches of the mathematicians are included.

"Newtonia," a set of ten portraits and other pictorial items relating to Newton, is also available from Scripta Mathematica.

The Signal Corps has prepared a series of posters, "Mathematics Goes to War Through the Signal Corps." Illustrative of the material found on these posters are the following titles: "Radicals and Exponents Enter into Circuit Problems," "The Mathematics of Radio Slopes,"

"Complex Quantities in Electricity," "Imaginarities and How They are Used in Electrical Circuits," "Graphs: the equation, the graph, the table," "Algebra and Electricity." These posters may be secured from Signal Corps Schools, 6th Service Command, 20 N. Wacker Drive, Chicago, Illinois.

Current magazines and newspapers are sources for diagrams and verbal statements related to algebra. As an example, the caption of Newsweek's October 1943 cover picture of Josef Stalin: "The X in the International Equation as the Moscow Conference Opens."

The extension of symbolism for daily use is exemplified in scores of ways in advertisements and daily application. The flying red horse is a trade symbol employed by a gasoline company; the letters LS/MFT are associated with a particular brand of cigarettes; a red and white striped pole is used extensively to indicate a barber shop. A red cross, the symbol of mercy, is displayed by many hospitals and ambulances. Motorists are given the reminder "It's time to retire" by a pajama clad youngster holding a lighted candle and standing by an automobile tire. American cartoonists have a more or less well developed store of symbols for depicting graphically, familiar persons, places and things. We are all well acquainted with the Democratic donkey and the Republican elephant. And what small child does not know what Uncle Sam symbolizes?

Material of this kind is invaluable for the development of concepts in the algebra laboratory.

The display of graphic materials is best accomplished through the use of bulletin boards. Well-lit, readable, within easy reach and view of the student, they become newsworthy and attractive sign boards for the posting of materials related to the activity at hand.

Students learn most when they participate directly in activities, and the bulletin board offers ample opportunity for participation. Individuals, committees, or the entire class may plan displays for the bulletin boards. General plans should be formulated for the type of material to be posted, when it goes up, and when it comes down. Particular features of the material may be indicated by underlining, or by colored strips of paper or ribbon extended from the underlined item to a typed notation or picture forming another part of the display.

The arrangements should be limited to one idea, aimed at establishing interest, and at clarification. If a bulletin board is large, it may be divided into sections, each section contributing materials related to a single idea. For example, when the symbolism of algebra is being studied, a single bulletin board would include material related to the history of symbolic representation; another to current uses of symbols in daily activities. A third bulletin board might contain the symbols of arithmetic, designed to bring to the

students' attention their own previous use of symbolism.

Quite commonly classrooms are not equipped with enough bulletin board space. Yet provision for additional space is comparatively simple. Wall space may be covered with some kind of durable cloth, or bulletin boards may be made by the students in the school shop from press board, plywood, heavy linoleum, or something similar. They can be hinged at the sides and folded, or suspended in the room on hooks so that they can be taken down when necessary.

A file of materials, to which teacher and pupils jointly contribute will include materials collected over a period of time and classified for easy reference. Clippings and pictures may be mounted on black poster paper--black because the possibility of visible fingerprints is lessened--or they may be enclosed in celluloid holders, such as those made by Gaylord Brothers, Syracuse, New York.

Materials should be classified and filed according to major topics in algebra in order to facilitate bulletin board arrangements. Many teachers may find useful the complete and understandable directions for filing clippings found in the pamphlet "An Information File in Every Library." Written by Ovitz and Miller, it is issued by the Library Bureau Division of Remington Rand, Buffalo, New York.

EARLY SYMBOLS USED IN ALGEBRA

+ AND - SIGNS WERE USED
IN WAREHOUSES TO SHOW
DIFFERENCE IN WEIGHT ...

PLUS

\tilde{p} $\tilde{\gamma}$
 p_{in} p
 $+$
 \times

MINUS

\bar{m}
 m_{en}
 M
 $\frac{-}{\div}$
 \approx

EQUALITY

[1550
 || 1575
 \rangle
 \sqcap
 \approx 1557

THE SIGN \approx IS THE INVENTION
OF ROBERT RECORDE

Figure 10 - Bulletin Board Arrangement

Projection Materials: Films, Filmstrips, and Slides.

Projection materials serve to extend the boundaries of students' experiences. In algebra the amount of available material of this type is small. Two films of value in the laboratory are "The Origin of Mathematics" and "The Slide Rule."

"The Origin of Mathematics," a 16 mm. 11 minute sound film is available from the Bell and Howell Company, Chicago. This film depicts the various types of mathematical symbols and processes employed by a succession of ancient people: the cavedwellers, Egyptians, Babylonians, Greeks, Romans, Arabs, and Indians.

Another motion picture of no little interest to algebra is "The Slide Rule," a 16 mm. sound film produced by Castle Films, New York. Designed to instruct in the use of the slide rule, it shows the use of the B and C scales of the slide rule to calculate proportions and percentages; the calculation of squares and square roots, the placing of decimals after the square root is extracted.

Films in the classroom can give an understanding of relationships, they can increase certain meanings involving motion, and they can bring the past and the distant into the classroom. Motion pictures, however, should not be used if some other teaching material is more effective, or when other equally effective but less expensive teaching devices can be used. Films may be profitably used to

supplement another teaching device, but if there is a choice, choose the most effective one.³

The choosing of films for a specific purpose increases the teacher's responsibilities. Intelligent use cannot be made of them unless their content is known and is found to be related to the topics of study. To accomplish this a preview must be made before the use of the film is made. The effectiveness of the film's use is determined by student discussion after the showing.

To facilitate future use a card file of films proves helpful. A 3" x 5" card for each instructional film should contain a synopsis, your evaluation, together with comments for future use. These are placed in a file and indexed by title or content.

Filmstrips, less expensive than motion pictures, are, many times, as effective as a film. They are made up of a series of related pictures printed on 35 mm. motion picture film, and are projected as still pictures. They may be prepared in black-and-white, in natural color, or in color tints. Usually kept in small cans with the title on the top of the can (where it is easily visible), they require little storage space.

The Society for Visual Education of Chicago, in conjunction with many other well known concerns in the visual field has been producing for many years, carefully

³Edgar Dale, Audio-Visual Methods in Teaching, pp. 191ff.

planned and expertly photographed filmstrips in many varied fields. Two of their filmstrips related to algebra are "The Origin of Algebra," and "Basic Definitions of Algebra." They are described as "a new series presenting an interesting and understandable transition from arithmetic to algebra. Especially prepared for high school students, stressing the social utility of algebra as a language."

The Jam Handy Organization of Detroit has prepared a series of filmstrips, "Light on Mathematics," designed to develop mathematical concepts. Available in four kits, containing from three to eight filmstrips, they are arranged according to their relation to arithmetic, geometry, algebra, and graphs. Kit III--Algebra, contains six filmstrips: positive and negative numbers, ratio and proportion, exponents and logarithms, the arithmetic of algebra, equations and formulas, and problem analysis. Kit IV--Graphs is equally applicable to algebra, containing three filmstrips: Graph uses, plotting graphs, and analytic geometry (graphing of quadratic equations).

Slide projections offer the advantage of magnification. Easily and inexpensively made from any kind of glass, they supplement photographic and other projection materials with local applications.

"History of Algebra," a set of 25 discussional slides showing the major stages in the progress of algebra

through the ages around the world, is available from Professor L. C. Karpinski, University of Michigan, Ann Arbor.

The Bureau of Publications of Teachers College, Columbia University furnishes a list of titles of lantern slide negatives illustrating the growth of algebra. These slides are highly recommended and undoubtedly would add to an appreciation of the development of algebra.

Classroom construction of lantern slides involves little difficulty. Students take pleasure in making slides for class use, and interest in algebra will be enhanced by this activity. As valuable as purchased and borrowed lantern slides may be, they do not provide the learning activity, the knowledge and selection of subject matter nor, the subsidiary skills accrued from actually constructing slides.

The steps in the making of a lantern slide are simple. A piece of glass about $3\frac{1}{4}$ " x 4" is cleaned, and covered with a thin coating of a gelatin solution. The solution is made by mixing about $\frac{1}{4}$ teaspoon of dessert gelatin to a cup of hot water in a container large enough to cover a slide. Hold the slide by the edges and dip one surface into the solution. It will dry in a few minutes. When the glass is dry, the drawing of a size to fit is placed under the glass and traced with India

ink on the dry gelatinized surface. Portions of the slide may be colored at this time, if desired.

Clear glass slides may also be written upon with a "china marking" pencil, sometimes called "ceramic" or "lithographic" pencil. It makes a rather heavy black mark, similar to crayon or crayola.

Plain lantern slide glass already cut to size is obtainable from commercial firms. Such glass is clearer and thinner than window glass, and the slides are uniform in size. Keystone View Company of Meadville, Pennsylvania furnishes for one dollar an instructor outfit containing materials and directions for slide making.

Etched glass slides are also used in slide-making. These too are available from Keystone View Company. Etched glass, the size of the regulation lantern slide, is placed over the drawing to be used and traced with an ordinary lead pencil. Lantern slide crayons may be used as the subject demands. This type of slide is good for free-hand sketches. Etched slides may be made from clear glass slides by dipping two clean slides in water, removing them and sprinkling the dampened surface of one side with No. F. Carborundum Powder. The other glass is placed over the powdered surface, and both glasses are rubbed together vigorously in rotary motion. When the surfaces on both slides are etched, place them under running

water to remove the carborundum powder. Let the slide dry well and it is ready for use.

A third possibility in lantern slide work, for the student interested in photography, is to make photographs on lantern slides. Glass, of the slide size, with one side sensitized for photographic work may be obtained from the Eastman Kodak Company. A complete account of the procedure is given with the package containing the plates. In this way slide pictures of algebraic models, instruments, or activities may be obtained.

Recreational Materials. Recreational materials, properly related to class work, are invaluable aids to teaching, and they present an aspect of mathematics the existence of which is oftentimes not even suspected.

The chief purposes of recreational materials are to enliven the subject of mathematics and to throw some light on the teaching of a concept, principle, or process. Such material may be playful, "just for fun," but it should not be mere fooling or trivial. It should have genuine mathematical value.

The materials may cover a wide range of topics drawn from the history of mathematics, the evolution and development of certain aspects of algebra, games, contests, puzzles, and applications to other subjects and fields

of activity. They are introduced into the classroom for stimulation and motivation purposes and they serve in many cases to sharpen and emphasize important ideas.

In his book "The Teaching of Mathematics," Schorling⁴ has presented a list of suggestions and illustrative materials for mathematical recreations. Another source of reference is found in Woodring and Sanford's⁵ book mentioned before. An extensive bibliography of recreational materials is given in an article which appeared in the February 1938 issue of School Science and Mathematics, entitled "Bibliography of Popular Mathematics."⁶ Another excellent bibliography and source list is given in the article "Enrichment Materials for First Year Algebra,"⁷ in the February 1939 issue of the Mathematics Teacher. Butler and Wren in "The Teaching of Secondary Mathematics"⁸ also present an extensive bibliography of recreational materials. This bibliography contains almost 250 titles which are classified under two headings, books and periodical references.

⁴Raleigh Schorling, The Teaching of Mathematics, pp.

⁵Woodring and Sanford, op. cit., pp. 85-89.

⁶D. B. Lloyd, "Bibliography of Popular Mathematics," School Science and Mathematics, XXXVIII (Feb., 1938), pp. 186-193.

⁷David W. Starr, "Enrichment Materials for First Year Algebra," Mathematics Teacher, XXXII (Feb., 1939), pp. 68-77.

⁸Charles H. Butler, and F. Lynwood Wren, The Teaching of Secondary Mathematics, pp. 119-128.

Organization of Materials. The advantages that supplementary materials offer to the laboratory are often non-existent if the materials are not immediately available for use at the time they are needed. A card catalog is essential for locating references and for keeping a check of the materials in the laboratory.

Pictures, clippings, and charts provide the most difficulty in filing. They are most easily located when filed according to specific subject headings. The arrangement of subject guides and folders allows for expansion at any time without disturbing the original material.

In determining the subject headings under which to file an article, it is essential to be uniform and consistent. Duplicate headings should be avoided; articles relating to one subject should not be distributed. Suggested headings for major divisions are: symbolism of algebra, formulas and equations, graphs, directed numbers, algebraic techniques to include materials related to such fundamental techniques as operations, factoring, fractions, exponents, and the like.

It is possible to sub-head these major divisions as much as is considered necessary for the purposes of the laboratory. However, filing like anything else, can be carried to extremes. Simplicity should be the keynote of any filing system that is to fit the needs of the students.

Individual records of pupils' achievements and evaluation lesson materials are kept in the teacher's personal files.

Sources of Materials. The impossibility as well as the impracticability of including all of the sources of materials which can be used in the mathematics laboratory is evident to the reader. The following list is, however, suggestive of the many kinds of materials which may be adapted to local situations and needs.

Instruments and Equipment

- Eugene Dietzgen Company, 218 E. 23rd Street, New York City.
Individual and blackboard equipment, pantographs, surveying instruments, graph paper of all kinds.
- Dennison Manufacturing Company, Framingham, Massachusetts.
Black lantern slide bindings (15¢ a roll), invaluable for making bar diagrams rapidly. Also red, green, and gold gummed paper ribbon (10¢ a roll), useful where several colors are needed for a graph.
- J. L. Hammett Company, 380 Jelliffe Avenue, Newark, New Jersey.
Graph Chart mounted on common roller, \$7.00; mounted on spring roller, \$8.50; painted on framed Hyloplate board, \$10.00.
- Harter School Supply Company, 2046 E. 71st Street, Cleveland.
The teacher's pantograph; uses chalk, crayon, or pencil; enlarges up to 60 inches; \$2.50. Pupil's pantograph: made of metal; 50¢ each; \$5.00 per dozen.
- Keuffel and Esser Company, 127 Fulton Street, New York City.
Individual and blackboard drawing equipment, surveying instruments of all types, pantographs, graph paper, and other materials. A Junior High School drawing kit, consisting of ruler, compasses, protractor, and graph paper is available.
- Yoder Instruments, East Palestine, Ohio
Manufacturers of individual and blackboard equipment, pantographs, surveying instruments, models, graph paper.

Reference Materials (Essays and Pamphlets)

Algebra Applied to Formula Analysis

"Algebra has a definite value that is generally overlooked in business figuring." Monroe Calculating Machines Company, Inc., 555 Mitchell Street, Orange, New Jersey. Free pamphlet. 1946.

Algebra of Four Thousand Years Ago

David Eugene Smith. Pamphlet available for 25¢ from Scripta Mathematica, 186th Street at Amsterdam Avenue, New York City.

Early Graphical Solutions of Polynomial Equations

Carl B. Boyer. Pamphlet available for 10¢ from Scripta Mathematica, 186th Street at Amsterdam Avenue, New York City.

"The Origin and Development of Algebra"

Louis Charles Karpinski. This article gives particular attention to the theoretical work of the Egyptians, but carries the work to the 19th century. School Science and Mathematics, XXII (Jan., 1923), 54-64.

Graphic Materials

High-lighted History of Algebra

Free wall-size chart available from Scott, Foresman and Company, 120 E. 23rd Street, New York City.

Illustrations Depicting the Use of Mathematics in Motor

Car Engineering. Educational Service Department, Chevrolet Motors Division, General Motor Sales Corporation, General Motors Building, Detroit, Michigan.

Mathematics Goes to War Through the Signal Corps

A series of posters dealing with topics of mathematics. Available from Signal Corps Schools, 6th Service Command, 20 N. Wacker Drive, Chicago, Illinois.

Newtonia

Ten portraits and other pictorial items relating to Newton, at 15¢ the item. Price of the whole collection, \$1.00. Available from Scripta Mathematica, 186th Street at Amsterdam Avenue, New York City.

Significant Temperatures

Free chart available from The Borden Company, Consumer Relations Division, 350 Madison Avenue, New York City. Contains problems to be solved on milk pasteurization on Fahrenheit and Centigrade thermometer scales.

Portfolio of Portraits of Eminent Mathematicians

Portraits suitable for framing, size 10" x 14".

Biographical sketches by David Eugene Smith are included. Available from Scripta Mathematica, 186th Street at Amsterdam Avenue, New York City.

Portfolio I: Archimedes, Copernicus, Viète, Galileo, Napier, Descartes, Newton, Leibnitz, Lagrange, Gauss, Lobachevsky, Sylvester. \$5.00.

Portfolio II: Euclid, Cardon, Kepler, Euler, Fermat, Chebycheff, Poincare. \$3.75.

Portraits purchased separately 50¢ each. Five or more, 35¢ each.

Projection Materials: Films, Filmstrips, Slides

The Origin of Mathematics

Film, 16 mm., sound. Rental \$1.50. Bell and Howell Company, 1801 Larchmont Avenue, Chicago 13, Illinois.

The Slide Rule

Film, 16 mm., sound. Castle Films, Inc., 30 Rockefeller Plaza, New York City.

Calculating Instruments; graphical methods, slide rules, and solving equations

Filmstrip. \$2.00. Society for Visual Education, Inc., 100 East Ohio Street, Chicago, Illinois.

Light on Mathematics, Kit III--Algebra; Kit IV--Graphs

Filmstrips, discussional. \$4.00. Jam Handy Organization, 2900 E. Grand Boulevard, Detroit, Michigan.

Origin of Algebra

Basic Definitions of Algebra

Filmstrips. \$2.00 each. Society for Visual Education, Inc., 100 East Ohio Street, Chicago, Illinois.

History of Algebra

Twenty-five discussional slides. \$22.00. Professor L. C. Karpinski, University of Michigan, Ann Arbor, Michigan.

Portraits of Mathematicians

Slides. Fifty-five cents each. University Prints, 11 Boyd Street, Newton, Massachusetts.

The Growth of Algebra

Slides. Eighty cents each in orders of 6 slides or more. Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York City.

Recreational Materials

"Socrates Teaches Mathematics"

A play showing the development of $(x+y)^3$ and $(x-y)^3$, using blocks to illustrate the various items. Norman Anning. School Science and Mathematics, XXII (June, 1923), 581-84.

"The Adventures of X"

A play dealing with the land of algebra. Maryl Clark, Wallace A. Newlin, Arthur E. Smothers. Boston: D. C. Heath and Company, 285 Columbus Avenue. Forty-eight cents.

"The Evolution of Numbers"

A historical drama in two acts. It offers excellent treatment of the reluctance with which negative numbers, zero, fractions, irrationals, and complex numbers were received into the number system. Its aim is to instruct as well as to amuse. Herbert E. Slaught. American Mathematical Monthly, XXXV (March, 1928), 146-151.

An Algebra Baseball Game

A recreational activity in algebra. Anonymous. Mathematics Teacher, XXIII (May, 1930), 317-320.

Número

A game of mathematical anagrams in which equations are formed from numbers, using signs of operation, parentheses, and exponents. Moffatt G. Bryce, 2619 Hampshire Road, Cleveland Heights, Ohio. Thirty cents each, four for \$1.00.

Amusements in Mathematics

Includes puzzles from algebra. Henry E. Dudeney. New York: Thomas Nelson and Sons, 381 Fourth Avenue. 1917. \$1.50.

Mathematical Nuts

Includes interesting and stimulating problems in algebra. Samuel I. Jones, 1104 Caldwell Lane, Nashville, Tennessee. 1932. \$3.50.

Mathematical Recreations

Chapter XIV of The Teaching of Junior High School Mathematics includes a variety of algebraic recreations. David Eugene Smith and William David Reeve. Boston: Ginn and Company, 15 Ashburton Place. 1927. \$2.00

Mathematical Wrinkles

Includes algebraic problems and recreations, questions, quotations. Samuel I. Jones, 1104 Caldwell Lane, Nashville, Tennessee. 1929. \$3.00.

Puzzles and Curious Problems

Includes algebraic puzzles and their solutions. Henry E. Dudeney. New York: Thomas Nelson and Sons, 381 Fourth Avenue. \$1.50.

Recreations in Mathematics

Includes puzzle problems dealing with algebra. H. E. Licks. New York: D. Van Nostrand Company, 250 Fourth Avenue. 1921. \$1.50.

The Canterbury Puzzles and Other Curious Problems

Includes a collection of algebraic puzzles. Henry E. Dudeney. New York: Thomas Nelson and Sons, 381 Fourth Avenue. 1932. \$1.50.

Summary. In the past the definition of problems and the selection of data for the solution of these problems has, in many cases, been limited to a single reference, the textbook. Experience has shown that students should be provided with opportunities not only to recognize and define their own problems, but also to select and organize the data necessary to establish and verify conclusions. This discussion has been concerned with the equipment, materials, and sources of materials that will prove helpful in the solution of problems as they arise from experiences indicated in the study guides that follow.

CHAPTER IV

STUDY GUIDES--SAMPLES OF DIRECTED ACTIVITIES IN ALGEBRA

Experimentation is the keynote of the laboratory method. The laboratory classroom sets the scene for a variety of worthwhile and enlightening experiences, yet there must be some arrangement to direct and guide the activity of the student if he is to make the most effective use of his time. Effective teaching must be based on accurate and well-defined objectives of study. Without clearly defined aims and purposes, instruction becomes ineffective, laboratory work tends to degenerate into more or less aimless playing, and students' efforts are poorly directed.

Study guides, planned in advance, are indispensable in this respect. Their chief aim is to direct pupils' activities and experiments, and to help them generalize from their experiences.

The organization and classification of study guides involves some attention to organization. Care should be taken to keep their content up to date since this increases their reality for the student. To insure their availability for immediate use they should be catalogued and filed according to general main headings. The guides within these groups can be subdivided

alphabetically according to their titles. The study guides within these groups, if further classification is necessary, are arranged in their probable order of use.

The study guides in this section have been arranged according to four main divisions: formulas, graphs, equations, and directed numbers. No complete subdivision is included. The study guides included under the main divisions are indicative of the types that may be developed.

The expected outcomes for which these materials have been arranged are listed below. Through these planned experiences students should be helped:

1. To appreciate the concise language of algebra and its use in expressing relations briefly and accurately
2. To discover the general law from an examination of particulars
3. To look for significant facts and relationships that may be revealed by graphs
4. To gather, organize, and express graphically statistical data of their own
5. To appreciate the importance and social significance of directed numbers

It should be noted however that these guides do not suggest completeness of content. They are samples of activities to illustrate how a few of the principles and concepts of algebra may be developed through the laboratory method.

I. FORMULAS

The formula is the theme about which a great deal of the work in algebra can be organized. Butler and Wren¹ point out that it involves and is closely associated with a great many of the concepts of elementary algebra; the symbolic language of literal numbers; constants and variables; the concept of dependence and function; graphic representation of relationships; substitution and evaluation; and operations with signed numbers, literal numbers, parentheses, exponents, fractions, radicals, and the like. Thus it forms a core which has points of contact not only with the previous experiences of the students but with many of the topics to be subsequently considered.

Materials and activities relating to algebra should be organized and planned to contribute to students' knowledge and understanding of

the meaning of dependence and of the relationship of independent and dependent variables

the ability to set up simple formulas expressing relationships existing in experiments and within the experience of the student

the expression of a relation between quantities by a table, a rule, a formula, and a graph

the substitution and evaluation of formulas.

In the study guides following, attention is given to the ways by which students may derive from their own experiments the formulas of introductory algebra.

¹Butler and Wren, op. cit., p. 274.

A. Deriving the Formula for the Perimeter of a Rectangle

If you were responsible for finding the amount of wire fencing needed to enclose the tennis courts at your school, you would be able to lessen considerably the number of measurements you would have to make if you knew the formula for the perimeter of a rectangle.

1. From the equipment cupboard take several wooden rectangles and a ruler. Measure the length of the two longer sides and the width of the other two sides. Record your measurements in the table below. Recalling the definition for the perimeter of any figure, what will you do to find the data for the last column of the table?

Perimeters of Rectangles					
	Length	Length	Width	Width	Perimeter
First Rectangle					
Second Rectangle					
Third Rectangle					
Fourth Rectangle					
Fifth Rectangle					

2. Is there any definite relation between the perimeter of a rectangle and its length and width? How many lengths did you add? How many widths?

3. What rule can you write for finding the perimeter of a rectangle when you know its length and width?

4. Look at the small rectangle shown here. It is how many units long? How wide is it? Let us use p to stand for the number of units in the perimeter. How can you now write the rule you stated in question 3?

5. You have written the formula for finding the perimeter of a rectangle. The formula is just a short way of writing the rule. What do we use to stand for numbers when we write a formula?

6. Are you able to find the perimeter of any rectangle, no matter how large it is?

7. If the length or the width of a rectangle increases, what happens to the perimeter? How does a decrease in the length or the width affect the perimeter?

8. Complete this statement: The perimeter of a rectangle depends upon its _____ and its _____

B. Deriving the Formula for the Area of a Rectangle

1. Cut from cardboard a rectangle 3" long and 2" wide, and several one-inch squares. Recalling the definition of area, how could you find the area of this rectangle by using the one-inch squares? Would you cover the surface of the rectangle with the squares? How would you then discover the area?

2. Cut out 4 more rectangles of the sizes indicated below:

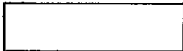
- (1) 5" long and 3" wide
- (2) 6" long and 2" wide
- (3) 4" long and 3" wide
- (4) 4" long and 4" wide

By experimenting with the rectangles and the one-inch squares you have cut out, complete the following table.

Rectangle	Number of times the square unit can be placed along the length of the rectangle	Number of times the square unit can be placed along the width of the rectangle	Number of times the square unit can be placed on the surface of the rectangle

3. Looking at the results in your table, do you see any way to find the area of a rectangle without measuring it directly with square units? Write a formula for the area of a rectangle.

4. Let's take a look now at your rectangle which is 6" long and 2" wide. In the table below, you can see that we may draw a picture of this rectangle, and indicate its length, width, and area. To find out how the area of a rectangle is affected as the width changes and the length remains the same, cut out a rectangle with length 6" and width 3". Cut out other rectangles with the dimensions: length 6" and width 4"; length 6" and width 5"; length 6" and width 6". Referring to your rectangles, complete the following table.

Picture	Length	Width	Area
	6"	2"	12 sq. in.

5. How is the area affected if the length is left unchanged and the width of the rectangle is increased?
6. Explain how the area would be affected if the width is left unchanged and the length of the rectangle is increased.
7. How is the area affected when both the length and the width steadily decrease in value?
8. The area of a rectangle depends upon its _____ and its _____.

C. Deriving the Formula for the Area of a Parallelogram

1. Construct from heavy cardboard a parallelogram, such as ABCD shown below. From D draw a line to AB, making it perpendicular to DC.

2. Next, cut off the small triangle thus formed, and place it on the opposite end of the parallelogram. The parallelogram has now been changed into a _____



3. Has the height of the parallelogram changed? Has the length of the base been changed? Has the area changed?

4. What does this experiment tell you about the area of a parallelogram and the area of a rectangle having the same base and height?

5. How do you find the height of a parallelogram?

6. What rule can you write for finding the area of a parallelogram?

7. Write this rule as a formula.

D. Deriving the Formula for the Circumference of a Circle

1. Secure three or four circular wooden discs from the equipment case. Measure the diameter of each very carefully. Avoid awkward fractions by using a ruler ruled to inches and tenths of an inch. Record your measurements in the table below. Next, to find the circumference of a disc, mark a point on the edge of the disc. Place this point on a straight line, and roll the disc along the line until the point touches the line again. Mark this point, and measure the length of the line. Why does this experiment give the circumference of the disc?

2. To find how many times as long as the diameter, the circumference is, you must find the ratio of the circumference to the diameter. What is a ratio? What is meant by an average ratio? How would you find the ratio of the circumference to the diameter? Find this ratio for the discs you have measured. Carry your division to two decimal places and record your answer in the table.

	1st Disc	2nd Disc	3rd Disc	4th Disc
Diameter (in inches)				
Circumference (in inches)				
Circumference divided by diameter (in inches)				

3. Find now the average of the ratios you have recorded in the table.

4. Looking at the results of your experiments, complete this statement: The circumference of a circle divided by the diameter is about _____

5. About how many times the diameter is the circumference of a circle?

6. Are you able to write the formula for finding the circumference of a circle when you know the diameter?

7. Does the formula show the relation that always exists between the circumference and the diameter of a circle?

8. In the formula for the circumference, the value of c depends upon the value of _____

9. The larger we make d in the formula, the _____ the value of the circumference will be.

E. Deriving the formula for the sum of the angles of a triangle.

1. Experiment with an adjustable triangle from the equipment case. Set the triangle in any desired position. Next read the measurements of the angles from their protractors.

2. Record your readings for several of your experiments in the table below.

	1st Reading	2nd Reading	3rd Reading	4th Reading
Angle 1				
Angle 2				
Angle 3				
Sum of the Angles				

3. In your table, record the sum of the angles for each triangle you formed when you changed the position of the sides of the triangle.

4. What is the average of the sums you have recorded?

5. From your observations, what would you conclude that the sum of the angles of a triangle is equal to?

6. As any one angle of the triangle increases in size, what happens to the other two?

7. Is it ever possible for two of the angles to be equal in size? When?

F. Finding the Area of a Circle

1. From the equipment case take one set of circle sectors and make sure that they fit together to form a circle.

2. Now spread out these sectors so that they are aligned as shown below.



3. What plane figure does each sector resemble? What is the common altitude of these figures? What is the base?

4. If you now take half of these sectors and fit them into the other half, the circle has been changed into a figure that resembles a _____

5. Suppose that we were able to divide a circle into millions of sectors and to spread these sectors out as we did above, would the figure formed be very different from a parallelogram?

6. What would be the altitude of this figure? What would be the length of the base?

7. You know that the formula for the area of any parallelogram is _____. Let us substitute the base and the altitude of this parallelogram we've just formed into the formula. We have A _____

8. You have just derived one formula for the area of a circle. But there is another formula for the area of a circle that is used even more commonly than this one. You have learned that $c = 2\pi r$. Suppose you substitute this value for c in your formula in question 5. How can you write the formula for the area?

9. Can you find the area of a circle using either one of these formulas? Will the area be the same in each case? Why?

10. What relationship exists between the area and the radius of a circle?

11. What happens if the radius changes?
12. If we increase the radius what happens to the area?
13. The area of a circle is dependent upon _____

The impossibility of including study guides to cover all phases of formulas, or any other topic of algebra for that matter, is evident to the reader. The guides included here attempt to illustrate the type of activities that lead students to think through their problems and to carefully analyze and verify their conclusions. Brighter pupils will undoubtedly require less guidance, but slower pupils may, on the other hand, demand a great deal more guidance. That is a matter of individual concern to the teacher. Study guides serve their purpose if they cause the learner to see how easy and simple the significant principles of mathematics really are.

II. EQUATIONS

The experience in solving equations that students bring to algebra has been largely on the intuitive level. Confronted with the equation $3n = 12$ and required to find the number represented by n , he reasons that if 3 n 's make 12, then one n will have to be one-third of 12, or 4. These intuitive reactions are sufficient and satisfactory for his purposes so long as the situation is simple, but the moment a problem situation becomes so involved that he is unable to keep all the elements, and their proper relationships clearly in mind at the same time, his intuition breaks down. When this happens logic must take its place, and in such cases the only recourse is to more formal and

powerful tools for the analysis of problem situations. Such a tool is the algebraic equation.² For it gives the student the power to investigate relationships which would be too complex to be investigated successfully or easily without its aid.

Many textbooks emphasize an early memorization of the four fundamental operational axioms of equality in the solution of linear equations. Butler and Wren³ however, support Paul Ligda's⁴ earlier recommendations that the utilization of a single unifying principle applicable to the solution of all linear equations makes a far greater contribution to a students' genuine understanding and power of analysis than any number of special techniques of solution. Such a principle can be stated in terms like these: We can solve any linear equation in one unknown for the unknown if we first, undo the operations associated with x --that is, apply the processes which are the inverse of those performed on x ; and second, preserve the relationship--whenever any legitimate operation is performed on one side of an equation, the same operation must be performed on the other side of the equation.

This principle gives the student basis for the solution of linear equations without relying entirely

²Butler and Wren, op. cit., p. 280.

³Ibid., pp. 281f.

⁴Paul Ligda, The Teaching of Elementary Algebra, pp. 48ff.

upon intuition or upon the memorization of processes.

"It gives emphasis to the character of the equation and lends organization and generality to the solution . . . which, in turn, eliminate any necessity for developing special methods for the different forms."⁵

In order to help students to visualize what is meant by the phrase "preserve the relationship" when applied to an equation, the following study guide has been prepared.

Equations--The Laws of Operation

1. Obtain from the equipment case a balance (a pair of scales) and a set of weights. Experiment with this balance. How is it made? What do you notice about it? What happens when you depress one side of the balance? What device on the scale tells you when it is in balance?
2. Place a 4 oz. weight on one side of the scale. What happens? How can you bring the scale back into balance? Add a two oz. weight to the first weight. Now what must you do to bring the scale back into balance?
3. Experiment with the scale, adding weights to either side, remembering after each addition to bring the scale back into balance.
4. It will help us to think of an equation as being like a pair of scales.

A Pair of Scales

(1) A pair of scales is an instrument that tells when two weights are equal.

An Equation

(1) An equation is a mathematical statement that two number expressions

⁵Butler and Wren, op. cit., p. 282.

(2) What divides the scales so that there are two sides?

(2) What divides the equation so that there are two sides?

(3) What tells you when the left side of the scale balances the right side?

(3) What tells you that the left side of an equation must balance the right side?

(4) Place a book on one side of the scales balanced on the other side by a 1 lb. weight. (How much does the book weigh?) Now add an 8 oz. weight to the side of the scales on which you have placed the book. What happens to the balance? What must you do to the other side of the scales to regain the balance?

(4) If you have an equation such that $x=7$, and you add 2 to the left side of the equation, what happens to the balance? What must you do to the right side of the equation to regain the balance?

(5) If you are working with a pair of scales and wish to keep the balance you cannot add anything to one side without

(5) If you add a number to one side of an equation, you must add the _____

(6) If you have a bag of candy, together with a 4 oz. weight on one side of the scales balanced on the other side by a 20 oz. weight and a 4 oz. weight, how can you find the weight of the candy?

(6) If you have an unknown quantity a added to a number 4 on one side of an equation balanced on the other side by $20+4$, how can you find the unknown quantity?

(7) How were you able to keep the scales in balance?

(7) How were you able to keep the equation in balance?

(8) If you are working with a pair of scales and wish to keep the balance, you cannot subtract anything from one side without _____

if you are to preserve the relationship.

(8) If you subtract any amount from one side of an equation, you must _____

if you are to preserve the relationship.

(9) Suppose now you place on one side of the scales a weight, say 3 Oz., balanced on the other side by 3 oz. Double the weight on the left side. What must you do to regain the balance? If you triple the weight, what must you do to regain the balance?

(10) If you double the weight on one side of the scales, you must also _____

in order to preserve the relationship.

(11) Suppose you have several weights on one side of the scale balanced on the other side by two 8 oz. weights. If you take one-half the weight from one side of this pair of scales, then you must _____

in order to preserve the relationship.

(9) Suppose you have an equation $x = 5$, and you multiply the left side by 2, what must you do to the right side to regain the balance? If you have an equation $\frac{x}{3} = 4$, what must you do to both sides to get the equation $x = 12$?

(10) If you multiply one side of an equation by a number, you must also _____

in order to preserve the relationship.

(11) Suppose you have the equation $4p = 16$. If you take one-half the number on the left side of this equation, then you must _____

in order to preserve the relationship.

(12) If you divide the weight on one side of a pair of balanced scales, you must also

(12) If you divide one side of an equation by some number, you must

in order to preserve the relationship.

in order to preserve the relationship.

5. The four laws of the equations which you have just studied may all be combined into one practical rule for solving equations: Whatever operation you perform on one side of an equation, you must also

6. What would you say is meant by this statement: Whenever we perform operations on an equation we must preserve the relationship.

A most effective presentation of the process of "undoing the operations" may be patterned after a demonstration performed by Professor Fawcett⁶ in one of his summer classes. A small box, wrapping paper, and string are placed on the demonstration table. An unknown object is held in the Professor's hand. Upon the blackboard an unknown quantity x is written. Several students are asked in turn to perform operations on this unknown quantity. Remarks such as "multiply by 3", "add 2", "multiply the whole quantity by 8", "form an equation of condition by placing the general number equal to 18", could yield

⁶Harold P. Fawcett, Lectures, Education 762, Ohio State University, Summer, 1947.

an equation similar to the one below:

$$8(3a + 2) = 18$$

As each suggested operation is performed upon the equation, an operation is performed upon the unknown object, finally resulting in a neatly wrapped and securely tied box.

AN UNKNOWN NUMBER

- (1) The unknown number

a

- (2) Multiply by 3

$3a$

- (3) Add 2

$3a + 2$

- (4) Multiply the whole quantity by 8

$8(3a + 2)$

- (5) Form an equation of condition by placing the general number equal to 18

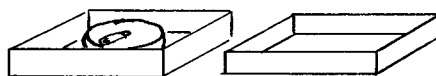
$8(3a + 2) = 18$

AN UNKNOWN OBJECT

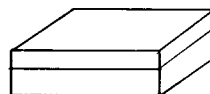
- (1) The unknown object



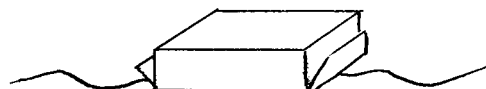
- (2) Place the unknown object in a box



- (3) Close the box



- (4) Wrap the box



- (5) Tie the string



The procedure of applying the inverse of the operations performed on the number in order to evaluate the unknown quantity achieves a high degree of reality when contrasted

with "undoing the operations" performed on the unknown object in the box.

THE EQUATION

(1) The equation

$$8(3a + 2) = 18$$

(2) Undo the operation by 8

$$\frac{8(3a + 2)}{8} = \frac{18}{8}$$

$$3a + 2 = \frac{18}{8}$$

(3) Undo the operation by 2

$$3a + 2 - 2 = \frac{18}{8} - 2$$

$$3a = \frac{1}{4}$$

(4) Undo the operation by 3

$$\frac{3a}{3} = \frac{\frac{1}{4}}{3}$$

(5) The unknown number

$$a = 1/12$$

THE BOX

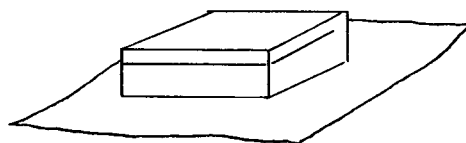
(1) The box



(2) Untie the string



(3) Unwrap the box



(4) Open the box



(5) The unknown object is a steel tape

The complete reasonableness of an equation is made apparent to the child immediately. To arrive at x simply apply the inverse of those processes which associate the letters or numbers to x. Though eventually the process of solving an equation will stand out as an abstract, mechanical principle of operation, it will not be devoid

of meaning because it has been built upon meaning in the beginning.

This demonstration is not experimental in the sense that the student himself performs the experiment, yet it does offer the concreteness necessary to give greater reality and meaning to a mathematical abstraction.

III. GRAPHS

The increasing prominence of graphs in newspapers, magazines, and other current magazines brings to students' attention the practical importance of the graph for picturing a relationship. Many and varied illustrations can be brought into the classroom for discussion and interpretation. Students will find within their own experiences many situations involving relationships among variable quantities that may be appropriately subjected to graphical treatment.

An understanding of the distinction between functional and statistical graphs, their nature and significance are fundamental to a comprehensive grasp of the idea that the graph is a picture of a relationship. The use, construction, and interpretation of functional graphs will receive its greatest attention in conjunction with the study of the formula. As an illustration, a graph of the relationship, $c = \pi d$, is best considered when the student is himself collecting the data and making a table of the values he ascertains. This provides an excellent situation for emphasizing the dependence of the

variable, the circumference c , upon another variable, the length of the diameter d , the value of π remaining constant. This formula $c = \pi d$ can then be used as a basis for constructing a graph. After the graph has been made, it should be carefully reexamined with attention directed particularly to the way in which it answers such questions as:

- (1) What happens to the circumference as the length of the diameter increases?
- (2) As the circumference increases, does the length of the diameter increase in the same ratio?
- (3) Does a decrease in either the circumference or the diameter bring a corresponding decrease in the other?
- (4) Explain how the graph shows that the circumference depends upon the length of the diameter.

In like manner, a study guide might be set up as follows:

A. Interpretation of Graphs

1. You no doubt know that the postage rate for air mail delivery to any place in the U. S. is five cents for each ounce or fraction of an ounce. If you let the cost be represented by c and the number of ounces by n , what formula can you write for the cost of sending letters by air mail?

2. Using this formula as a basis, complete the table below. Then on a sheet of squared paper make a graph to show the relation between the cost and the weight of an air mail letter.

Weight (in ounces)	1	2	3	4	5	6	7	8
Cost (in cents)								

3. Refer to your graph to answer the following questions.

- (1) What happens to the cost as the weight (number of ounces) of the letter increases?
- (2) As the cost increases, does the number of ounces increase in the same ratio?
- (3) Does a decrease in either of the variables bring about a corresponding decrease in the other?
- (4) How does the price per ounce affect the direction of the graph?
- (5) During the war years the price per ounce for an air mail letter was 8¢ instead of 5¢. How does an increase in price change the formula? Graph this relationship on the same axes as your first graph. How is the direction of the graph changed?
- (6) Would the selection of different scales for numbering the axes cause the direction of the graph to be different from its present direction?
- (7) Explain how the graph shows that the cost depends on the number of ounces the letter weighs.

Statistical graphs occur almost daily in newspapers, and they are quite commonly found in business magazines. Students themselves should be encouraged to bring in graphs that interest them. Class discussion of these graphs clarifies their meanings and provides a basis for understanding other graphs of similar type.

The following guide is arranged around a clipping from a newspaper. It gives an indication of the potential opportunities in the use of this kind of material.

B. Interpretation of Graphs

Not long ago this line graph of statistical data concerning annual savings appeared in a local newspaper. Study this graph carefully and indicate your opinion of the validity of the statements that follow. If you consider the statement to be true, underline the word true; if you consider it to be

false, underline the word false. If on the other hand, you do not think the graph indicates whether the statement is true or false, underline the word, uncertain.

(1) The greatest annual savings of individuals were made during the year 1944.

True False Uncertain

(2) The greatest annual savings before 1941 were during the year 1935.

True False Uncertain

(3) Notice that the line through the points for the year 1936 and 1944 makes "peaks" for those years. It is correct to say that the annual savings of individuals reached a peak in 1944.

True False Uncertain

(4) There was a rapid and steady increase in the annual savings of individuals from 1940 to 1945.

True False Uncertain

(5) No annual savings by individuals were made during the years 1932 to 1934. In fact, individuals spent some of their savings that had been accumulated during earlier years.

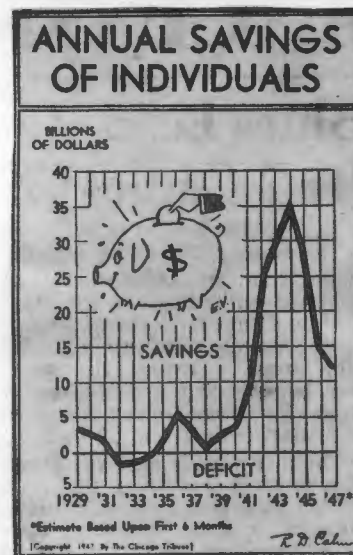
True False Uncertain

(6) The least annual savings by individuals were during the years 1932, 1933, 1934, and 1938.

True False Uncertain

(7) The annual savings of individuals will steadily drop during 1947.

True False Uncertain



Tables are another source of material for graphs. They appear in publications perhaps even more often than graphs do. Experience in interpreting tables develops abilities in comparing data. To illustrate, the following guide is centered around a clipping from an afternoon newspaper.

C. Construction and Interpretation of Graphs

Beneath the caption "Mercury Headed for New High" this table appeared in a local newspaper. Make a line graph of the data in this table and see if you can answer the questions that follow.

1. The table and the graph show the same facts. Which method makes it easier for you to see the temperature changes?

2. Can you tell from the graph what the temperature was at 9 a.m.? On what scale do you find 9 o'clock? On what scale do you read the temperature?

3. What was the temperature at 11 a.m.? at 1 p.m.?

4. At what hour was the temperature highest? When was it lowest?

5. Between what hours was the temperature always increasing? Between what hours was it decreasing?

6. When was the temperature 72°? When was it 75°? (Note that some discussion of the meaning of continuity in graphs is necessary here.)

7. The graph is made from the table. How could you make the table if you had only the graph to guide you?

HOURLY TEMPERATURES	
Temperatures in Columbus, beginning at midnight Monday, as recorded by the downtown U. S. Weather Bureau, follow:	
12 (midnight)	79
1 a. m.	79
2 a. m.	77
3 a. m.	76
4 a. m.	74
5 a. m.	72
6 a. m.	71
7 a. m.	73
8 a. m.	76
9 a. m.	80
10 a. m.	84
11 a. m.	86
12 (noon)	88

IV. DIRECTED NUMBERS

Pupils' numerical experience prior to the introduction of the concept of negative numbers has been confined almost entirely to numbers representing quantities greater than zero. Now, however, it becomes necessary to give zero a new significance and to regard numbers as being positively or negatively directed not only with respect to zero, but also with respect to each other.

Many educators advocate the use of a directed number scale to help the student gain an understanding of the meaning of directed numbers. To quote Butler and Wren, "The number scale is probably the most satisfactory and helpful of all devices for making clear the nature of positive and negative numbers and for illustrating their characteristics of oppositeness, direction, and position."⁷

The study guides that follow include experiences to help students acquire an understanding of the nature of negative numbers as contrasted with positive numbers, and to discover the rules for the fundamental operations with signed numbers.

⁷ Butler and Wren, op. cit., p. 293.

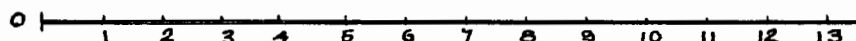
A. The Meaning of Directed Numbers

1. Take a Fahrenheit thermometer from the equipment case. Notice how the numbers that indicate the temperatures are arranged. Where is zero on the scale? What is the largest number on the scale? Are you able to count the numbers from 0 to 212 on the thermometer? In which direction from zero are these numbers? These numbers are the same familiar numbers of arithmetic and are called positive numbers.

2. Are there any numbers on the scale in the opposite direction from zero? What are they? These numbers counted in the opposite direction also have a name. They are called negative numbers to distinguish them from the positive numbers.

3. To see the difference between the numbers used in arithmetic and those used in algebra, let us consider the arithmetic scale of numbers and the algebraic scale of numbers.

The arithmetic scale represents a series of numbers which extend in only one direction from zero.



Draw a line from left to right on your paper. Divide it into equal parts. Select one of these division points as your zero point. You can now number your divisions in both directions from zero. Do you know of any way to distinguish the negative numbers from the positive numbers?

This point in the study of directed numbers offers an opportunity to guide pupils to references which discuss the historical significance and symbolism of negative numbers. A knowledge of the many early symbols used to distinguish negative numbers helps pupils to appreciate the arbitrary character of the selection of the minus sign.

4. The numbers used in algebra, like those you commonly see on a thermometer, extend _____ from zero on the algebraic scale.

5. Suppose we look at the scale you have drawn. By definition (note that there is opportunity here to develop the idea of the arbitrary character of definitions) the positive direction on the scale is to the right, and the negative direction is opposite the positive. Referring to your scale, complete the table below.

With reference to 0	+3 is in the <u>positive</u> direction because +3 lies to the <u>right</u> of zero.
With reference to +7	+3 is in the <u>negative</u> direction from +7 because +3 lies to <u>left</u> of +7.
With reference to -2	-1 is in the <u>positive</u> direction from -2 because -1 lies to the _____ of -2.
With reference to -4	-9 is in the _____ direction because _____
With reference to +3	-2 is in the _____ direction because _____
With reference to -1	0 is in the _____ direction because _____
With reference to -5	+2 is in the _____ direction because _____

6. From your observations you are now able to judge the validity of these two statements. Mark each statement True, False, or Uncertain.

(1) A number is positive with reference to any number that lies to its left on the scale.

(2) A number is negative with reference to any number that lies to its right on the scale.

B. The Meaning of Directed Numbers

1. This table appeared in the financial section of a recent newspaper. It shows the standings of stocks on a Stock Exchange for one day. Taking Sunray Oil stock as an example, you can tell from the table that there were 37,500 shares of the stock sold that day; the price was \$11 $\frac{5}{8}$ per share; this closing price was a gain of $\frac{1}{8}$ of a point over the closing price of the day before; that is, it was \$1 $\frac{1}{8}$ or 12 $\frac{1}{2}$ ¢ higher.

Stock Exchange Leaders			
	Volume	Last	Net Chge.
Packard	125,500	5	— $\frac{1}{4}$
Comwlth & So.	100,700	3 $\frac{1}{4}$	— $\frac{3}{4}$
Param Pict ...	78,500	23 $\frac{1}{2}$	— 3 $\frac{1}{2}$
Am Woolen ...	69,800	44 $\frac{1}{2}$	+ 1
St L San Fran.	56,200	9 $\frac{1}{2}$	+ $\frac{1}{2}$
United Corp ..	54,300	3	— $\frac{1}{2}$
Loew's	52,000	19 $\frac{1}{2}$	— 2 $\frac{1}{2}$
Nat Cont'r new	50,200	14 $\frac{1}{2}$	+ $\frac{3}{4}$
Sunray Oil	37,500	11 $\frac{1}{2}$	+ $\frac{1}{8}$
Warner Pict ..	33,500	14 $\frac{1}{2}$	— 2
St L San Fr pf	31,500	31 $\frac{1}{2}$	+ 3 $\frac{1}{2}$
Socony-Vac	31,300	16 $\frac{1}{2}$	— $\frac{1}{2}$
Int Nickel	29,300	29 $\frac{1}{2}$	— 1 $\frac{1}{2}$
Chrysler	28,400	57	— 2
Twent Cen Fox	26,400	26 $\frac{1}{2}$	— 3 $\frac{1}{2}$

2. A net change of + $\frac{5}{8}$ means that the price of a share 62 $\frac{1}{2}$ ¢ over the price at the close of the previous day.

3. What does a net change of - $\frac{1}{4}$ mean?

4. In the table, + means increase or gain. It follows therefore that - means _____

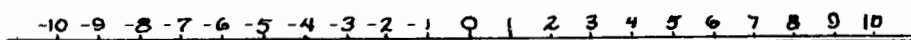
5. Of the 15 stocks listed, _____ showed a loss for the day, _____ showed a gain, and _____ showed neither loss nor gain.

6. Tabulate the amount of gain or loss for each stock during the day's trading.

Name of Stock	Gain	Loss
Packard		1/4
Comwlth & So		3/8
Param Pict		3 7/8
Am Woolen	1	
St L San Fran		
United Corp		
Loew's		
Nat Cont'r new		
Sunray Oil		
Warner Pict		
St L San Fr pf		
Socony-Vac		
Int Nickel		
Chrysler		
Twent Cen Fox		

C. Adding Directed Numbers

1. The addition of positive and negative numbers may best be understood by considering the algebraic number scale.



You already know that the positive direction on the scale extends to the right and that the negative direction extends to the left.

2. Let us suppose you wanted to add $+3$ and $+5$. You can reason like this: I am at $+3$ on the scale; To add $+5$, I must count _____ units in the _____ direction; and I arrive at _____ which is my answer.

3. Suppose next you want to add $+7$ and -2 . At what point do you begin on the scale? In which direction do you count? How many units do you count? You arrive at what number?

4. In the table below indicate what you would do to find the sums of the following numbers.

Numbers to be added	Start at	Direction to count	Units counted	Sum
$+2$ and $+4$	$+2$	Positive	4	$+6$
$+7$ and $+1$				
$+3$ and $+8$				
-5 and -3	-5	Negative	3	-8
-7 and -4				
-7 and -1				
$+5$ and -8				
$+7$ and -2				
10 and -3				
-5 and $+8$				
-3 and $+1$				
-9 and 14				

5. Referring to the results in your table, how would you complete the following statements?

- (1) The sum of two positive numbers is a _____ number.
- (2) The sum of two negative numbers is a _____ number.
- (3) The sum of a positive and a negative number is the _____ between the two numbers, and the sign of the sum is the same as the sign of the _____ number.

D. Subtracting Directed Numbers

1. In subtracting positive and negative numbers let us again look at the number scale. If you wanted to subtract $+2$ from $+7$ you would reason: What number must I add to $+2$ to get $+7$. Where would you start on the scale? At what number do you want to stop? In which direction must you count to reach your stopping point? How many units have you counted from $+2$ to reach $+7$? What is the difference between $+2$ and $+7$?

2. Next, suppose you wished to subtract $+5$ from -3 . Where would you start on the scale? At what number do you want to stop? In which direction must you count to reach the number at which you're to stop? How many units have you counted from $+5$ to reach -3 ? What is the difference between $+5$ and -3 ?

3. In the table below, indicate what you would do to find the differences of the following numbers.

Numbers to be subtracted	Start at	Stop at	Direction to count	Units counted	Difference
$+7$ from $+3$	$+7$	$+3$	Negative	4	-4
-3 from $+5$	-3	$+5$	Positive	8	$+8$
-2 from -7	-2	-7	Negative	5	-5
$+2$ from -5					
-3 from $+8$					
-7 from -4					
-1 from $+5$					
$+4$ from -6					

4. Suppose now you complete these tables which contrast the results you obtained in your subtraction table and in your earlier addition table.

Subtraction	Addition
$+7$ from $+3 = -4$	-7 and $+3 = -4$
-3 from $+5 =$	$+3$ and $+5 =$
-2 from $-7 =$	$+2$ and $-7 =$

5. In the first example of each table, what difference is there between the number to be added to $+3$ and the number to be subtracted from $+3$? Is this same difference true of each of the other examples? To subtract one number from another, we could find our difference by adding if we did what one thing?
6. What generalization can you make about the subtraction of directed numbers?

E. Multiplying Directed Numbers

1. You know that multiplication is a short-cut for addition. Thus, $(+2)(+3)$, to be read $(+2)$ multiplied by $(+3)$, means $(+2) (+2) (+2)$. If we look at the directed number scale, we can readily see that $(+2) + (+2) + (+2)$ means that we must add $(+2)$ to zero three times.



If we add 2 to 0 one time, at what point on the number scale are we? If we add 2 to 0 two times, at what point on the number scale are we? If we add 2 to 0 three times, at what point on the number scale are we? What is the product of $(+2)(+3)$?

2. In like manner $(-2)(+3)$ means $(-2) + (-2) + (-2)$. On the number scale this means that we must add (-2) to zero three times. If we add (-2) to 0 one time, at what point on the number scale are we? If we add (-2) to 0 two times, at what point on the number scale are we? If we add (-2) to 0 three times, at what point on the number scale are we? What is the product of $(-2)(+3)$?

3. In the problem above, the sign of which factor told us to add? In the problem $(+2)(-3)$ what is the sign of the second factor? What does it tell us to do? The product of $(+2)(-3)$ means, therefore, $(+2) - (+2) - (+2)$. On the number scale this means that we must subtract $(+2)$ from zero three times.

When we subtract $+2$ from 0 one time, we must remember that subtraction is the inverse of addition. Therefore, we think: What number must be added to $+2$ to result in 0? If you remember how to add directed numbers, you answer -2 . At what point on the number scale are we when we subtract $+2$ from 0 one time? If we subtract $+2$ from 0 two times at what point on the number scale are we? If we subtract $+2$ from 0 three times at what point on the number scale are we? What is the product of $(+2)(-3)$?

4. The product of $(-2)(-3)$ means $(-2) - (-2) - (-2)$. On the number scale this means that we must subtract (-2) from 0 three times. If we subtract -2 from zero one time, we think: What number must be added to -2 to result in 0? Remembering how to add directed numbers you answer 2. At what point on the number scale are we when we subtract -2 from 0 one time? If we subtract -2 from 0 two times, at what point on the number scale are we? If we subtract -2 from 0 three times at what point on the number scale are we? What is the product of $(-2)(-3)$?

5. Experiment with several products before you complete the following tables.

Numbers to be multiplied	Number to be <u>added</u> to 0	Number of times the number is to be added to zero	Product
$(+2)(+3)$	+2	3	+6
$(+7)(+2)$			
$(+5)(+1)$			
$(+3)(+6)$			
$(-4)(+2)$	-4	2	-8
$(-6)(+3)$			
$(-2)(+3)$			
$(-1)(+5)$			

Numbers to be multiplied	Number to be <u>subtracted</u> from zero	Number of times the number is to be subtracted from zero	Product
$(+1)(-2)$	+1	2	-2
$(+3)(-7)$			
$(+5)(-4)$			

$(+2)(-3)$			
$(-9)(-2)$	-9	2	-18
$(-3)(-8)$			
$(-1)(-6)$			
$(-5)(-4)$			

6. By referring to your tables, you can complete these statements.

(1) If two positive numbers are multiplied, their product is a _____ number.

(2) If two negative numbers are multiplied, their product is a _____ number.

(3) If a positive number and a negative number are multiplied, their product is a _____ number.

F. Dividing Directed Numbers

1. In dividing positive and negative numbers, we remember that division is the inverse of multiplication. That is, in dividing 30 by 6, we may think: what number is multiplied by 6 to give 30. Our answer, of course, is 5.

2. Suppose we wished to divide -24 by 8. We think: what number is multiplied by 8 to give -24? Remembering the rules for the multiplication of positive and negative numbers, we are able to answer -3.

3. Reasoning in this manner, complete the following table.

Numbers to be divided	Quotient
+49 by +7	
+12 by +4	
+24 by +6	
-56 by -8	
-100 by -5	
-36 by -6	
+14 by -7	
+22 by -2	
+ 6 by -3	
-8 by +2	
-9 by +3	
-16 by +8	

4. Referring to your table how many of these statements can you complete?

- (1) If two positive numbers are divided their quotient is a _____ number.
- (2) If two negative numbers are divided their quotient is a _____ number.
- (3) If a positive number and a negative number are divided, their quotient is a _____ number.

CHAPTER V

SUMMARY

The primary objective of education in a democratic culture is the development of individuals capable of using clear and accurate habits of thought. Algebra, redefined in terms of this objective becomes worthwhile to the individual as it becomes for him a means for identifying the relationships of changing values in the world about him.

The problem in this thesis has been one of indicating materials and procedures to be used in helping students to develop understanding and insight into the basic concepts of algebra. The laboratory method of instruction provides an opportunity for the student to gain these understandings through his own direct experience.

The adaptation of the classroom to a mathematics laboratory provides a selection of equipment and other instructional materials for his use in investigating, collecting, and organizing data as aids to his development of reflective thinking. Of the many opportunities for experiences in the mathematics laboratory the study of algebra may help pupils

- to gain an enlarged understanding of basic concepts and principles of mathematics

- to gain a knowledge of the methods of expressing mathematical relationships, and an ability to use these methods

to develop habits of clear and accurate thinking,
to collect, organize, and identify data, and to
interpret the data so organized in terms of relations
that exist between them

to gain an understanding and appreciation of the
many applications of mathematics

to gain increased skill in using certain instruments
and devices

to develop an appreciation of the historical
significance of mathematics

In order to guide students toward the attainment of these outcomes, study guides have been designed to provide experiences from which students may be led to make generalizations of the principles and ideas of mathematics which they have discovered. To enrich experiences and to motivate students' study, sources of reference, recreational, graphic, and projection materials with suggestions for their use in the laboratory, have been included.

Throughout this study attention has been directed to the development of concepts and meanings through students' direct and active participation in experiments and activities leading to investigation and discovery of the basic relationships in algebra.

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